ITERATIONS

JoHN MiLLeR
Portland, OREGON
June 30, 2024

dialectrix.com / G4G

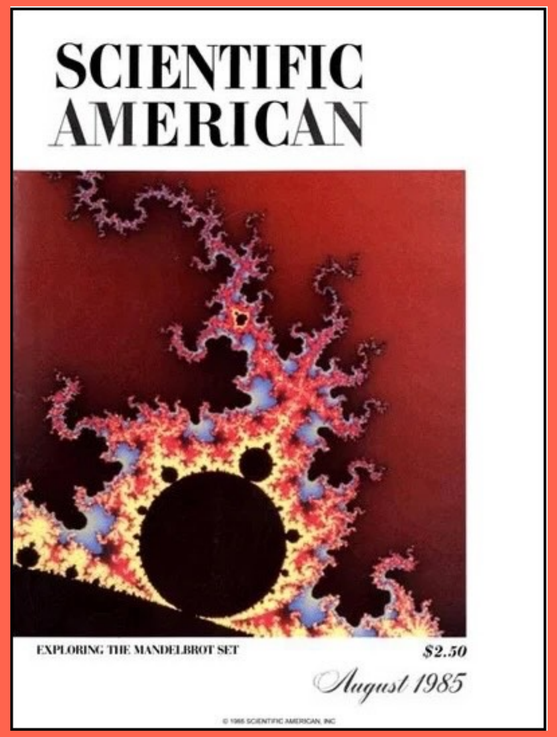
Scope Narrowed!

Two kinds of iteration,

with lots of graphics,

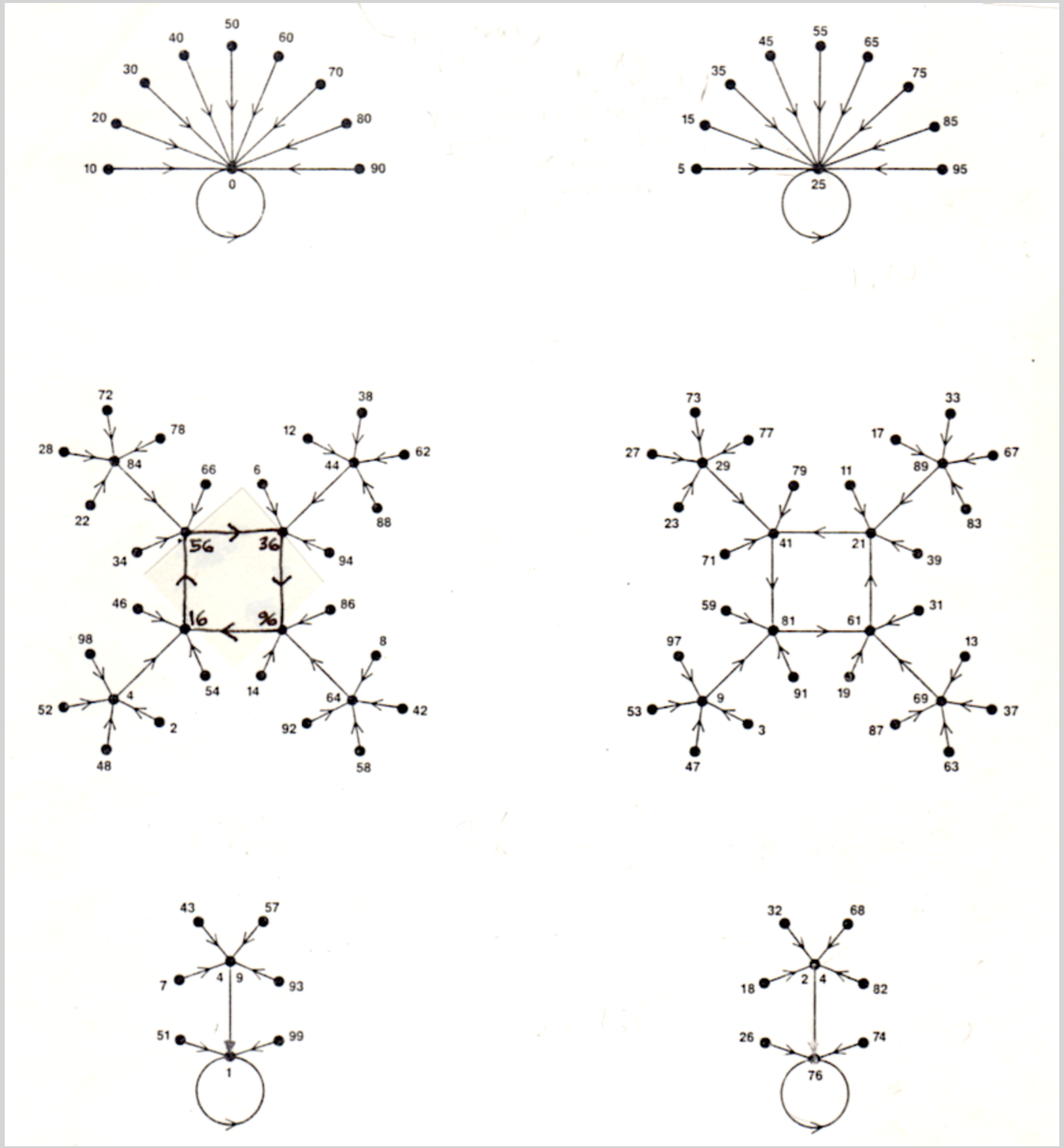
and a few digressions.

$$N = N + 1$$


SCIENTIFIC
AMERICAN
EXPLORING THE MANDELBROT SET
$2.50
August 1985
© 1985 SCIENTIFIC AMERICAN, INC

AUG 1985

Sci Am Aug 85 - *Computer Recreations*, page 21



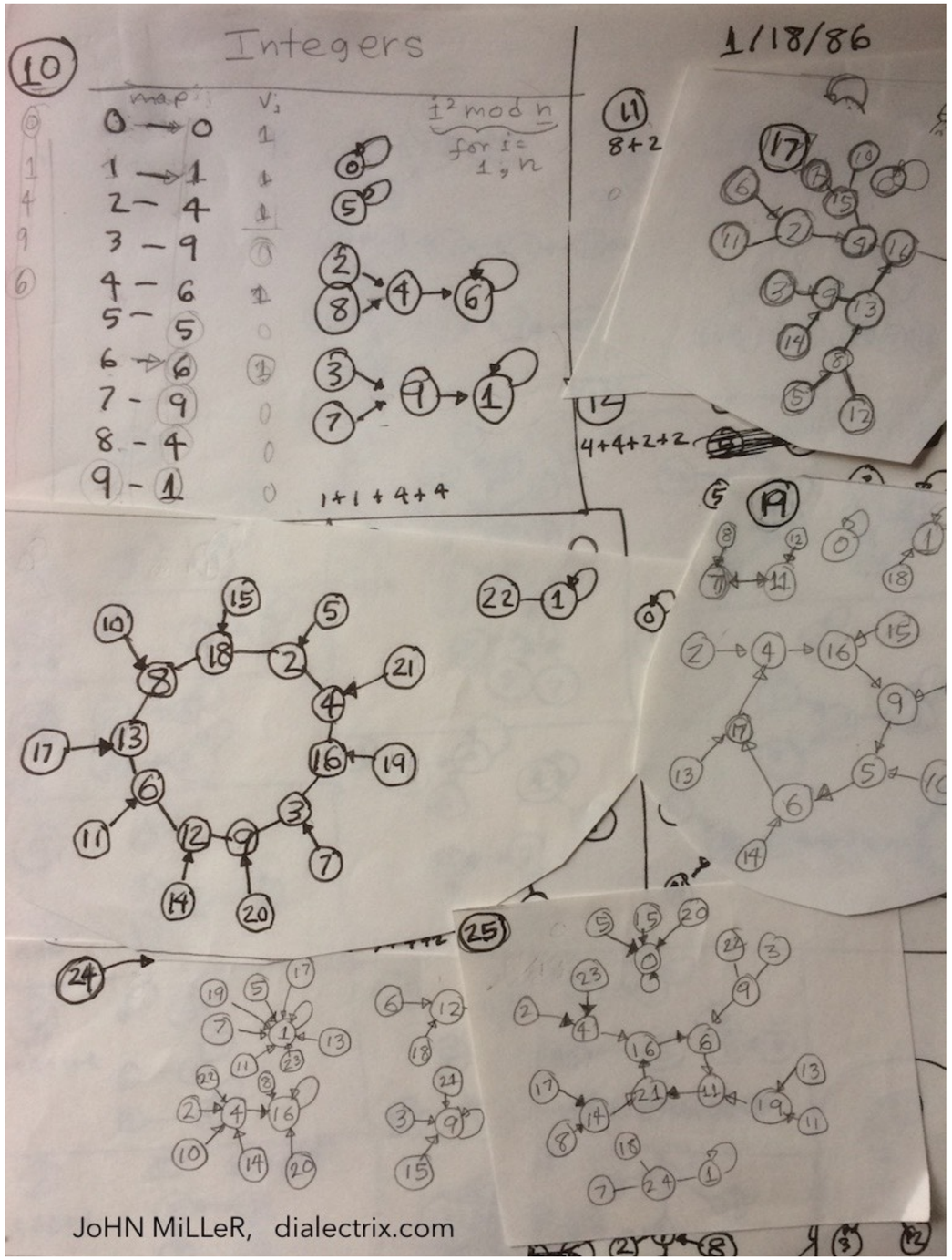
The six components of the iteration diagram
for squaring the first 100 integers

$i \to i^2 \bmod m$

EXAMPLE, modulo 10

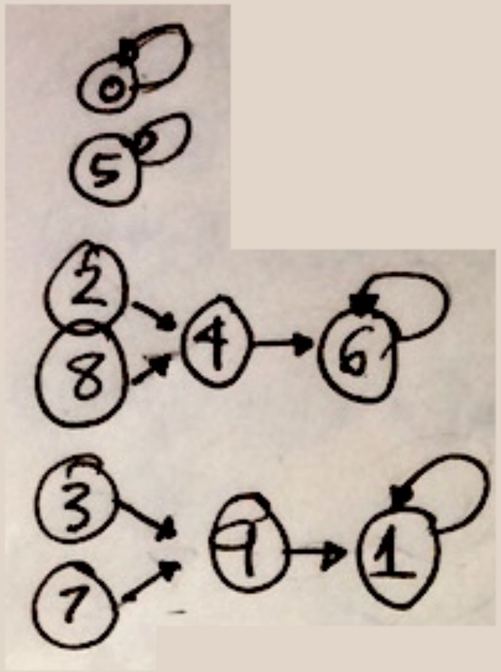
Start with $i = 9 \quad \to \quad 9^2 \bmod 10 \quad = \quad 81 \bmod 10 \quad = \quad 1$

We say… 9 ‘goes to’ 1


Integers
1/18/86
map
v_i
$i^2 \bmod n$
for i = 1, n
0 → 0
1 → 1
2 – 4
3 – 9
4 – 6
5 – 5
6 → 6
7 – 9
8 – 4
9 – 1
1+1+4+4
8+2
4+4+2+2
JoHN MiLLeR, dialectrix.com

10

0 → 0
1 → 1
2 → 4
3 → 9
4 → 6
5 → 5
6 → 6
7 → 9
8 → 4
9 → 1



We prepare input for

GraphViz - Neato

layout algorithm

which will automatically

generates a graph… in

PostScript, SVG, et al.

0 -> 0

1 -> 1

2 -> 4

3 -> 9

4 -> 6

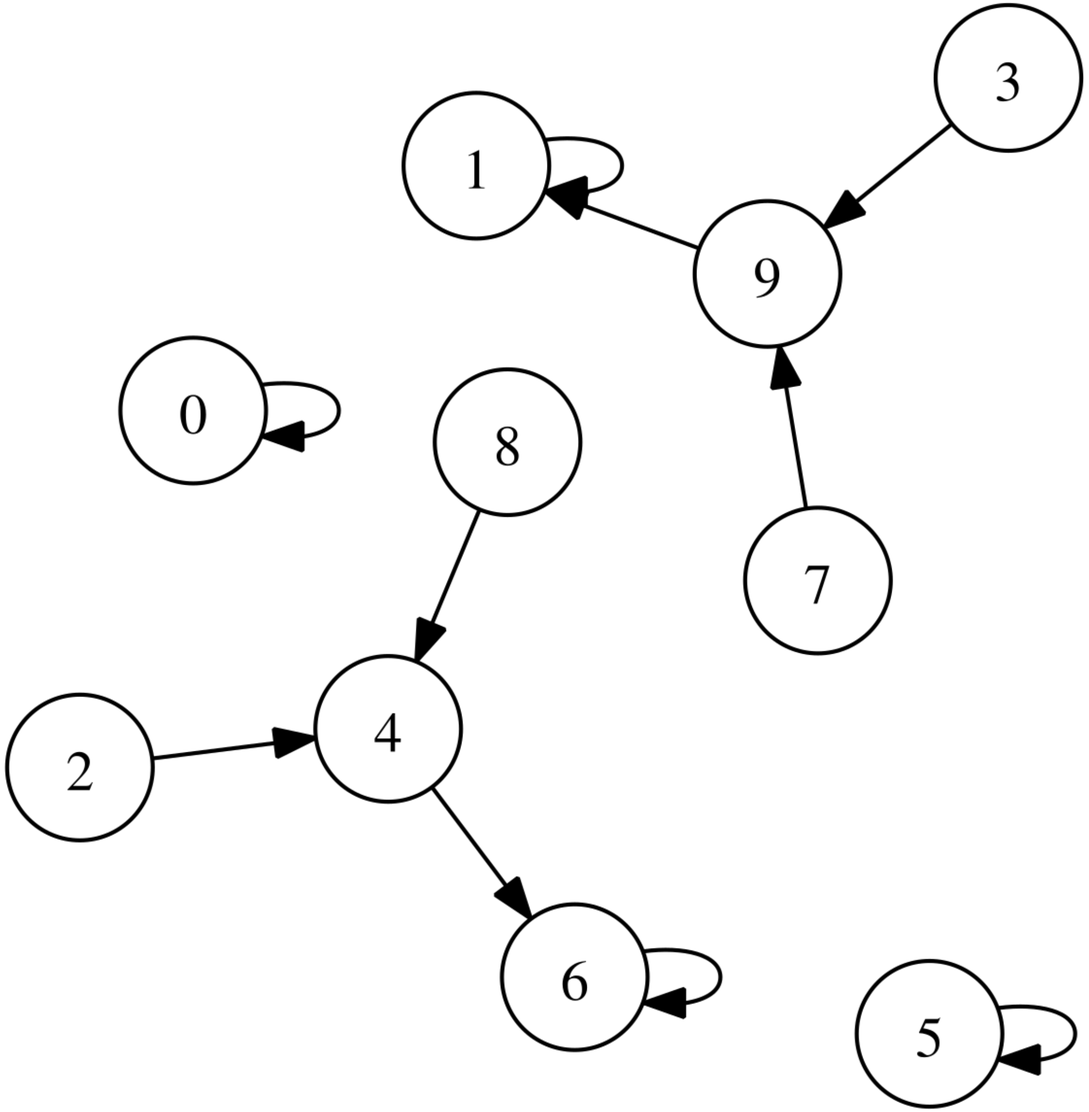
5 -> 5

6 -> 6

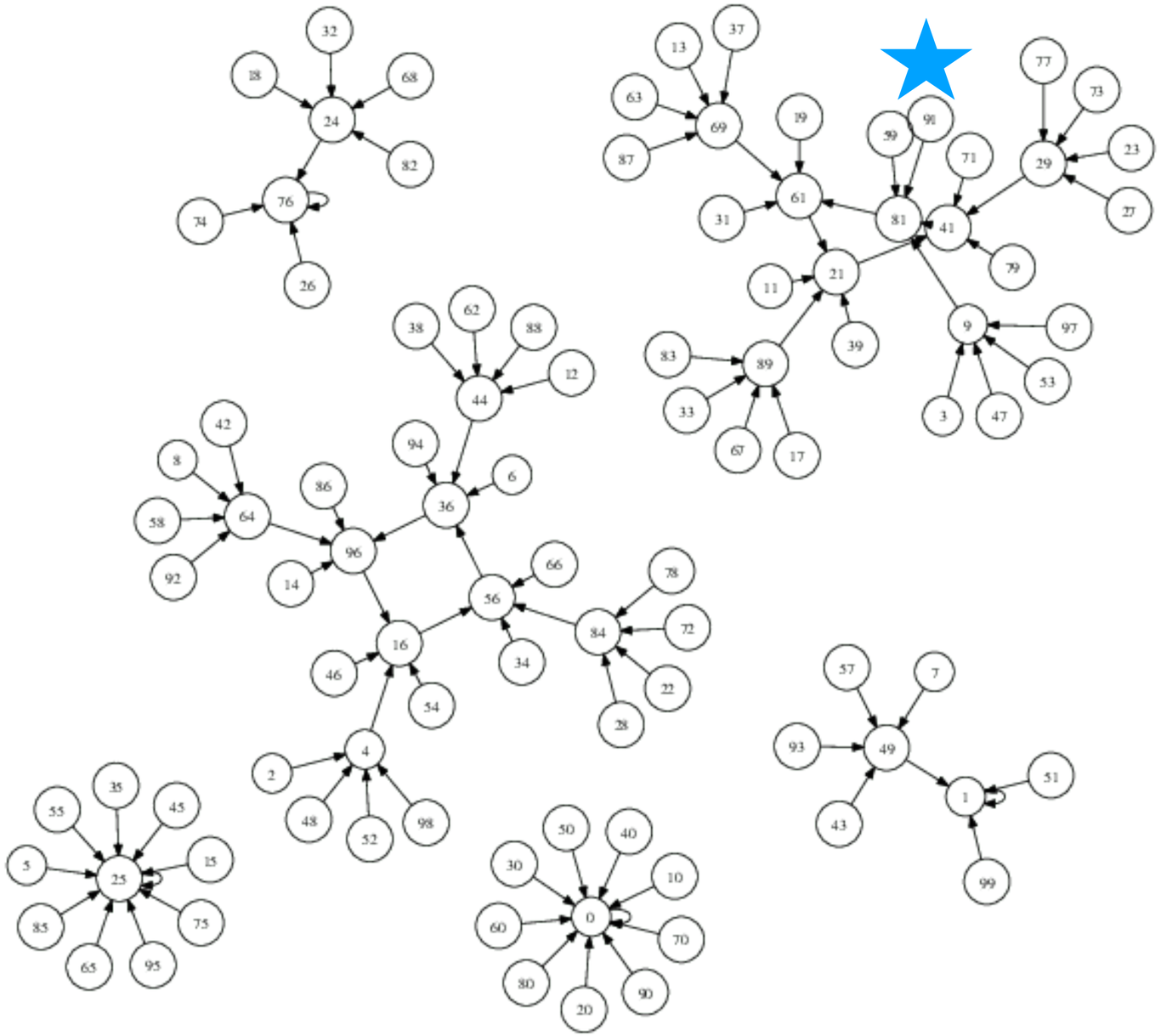
7 -> 9

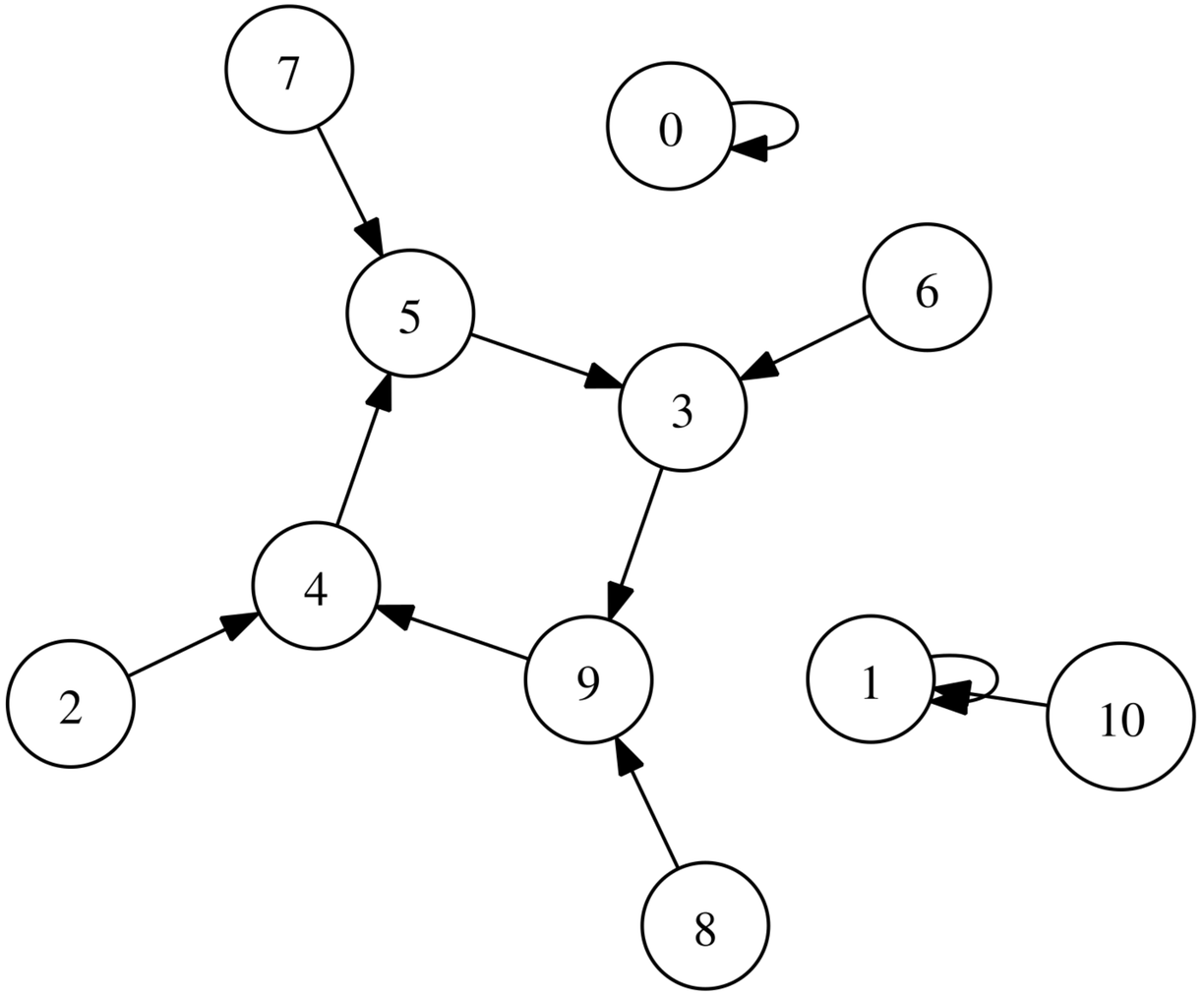
8 -> 4

9 -> 1

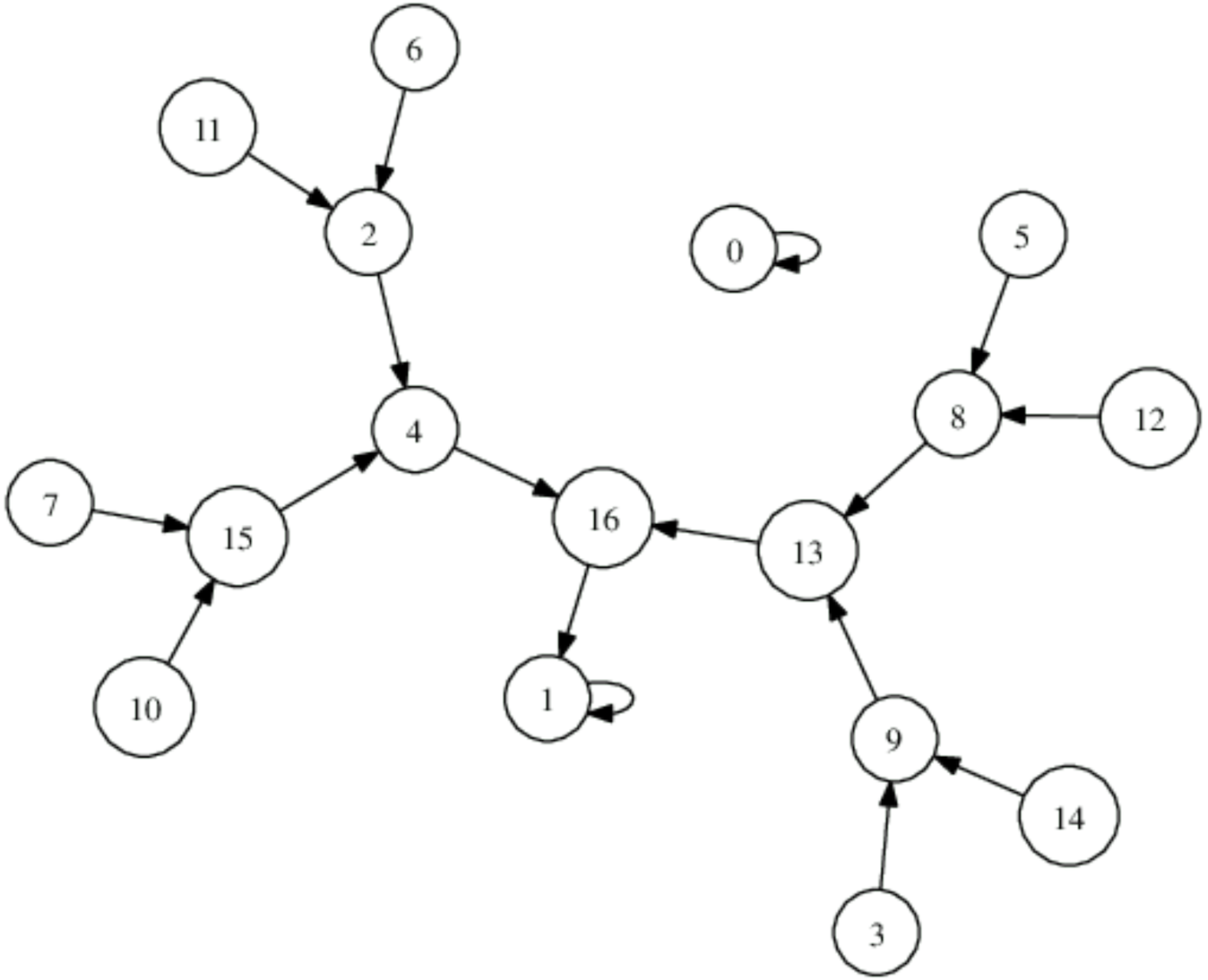


Isomorphic to my hand drawn graph

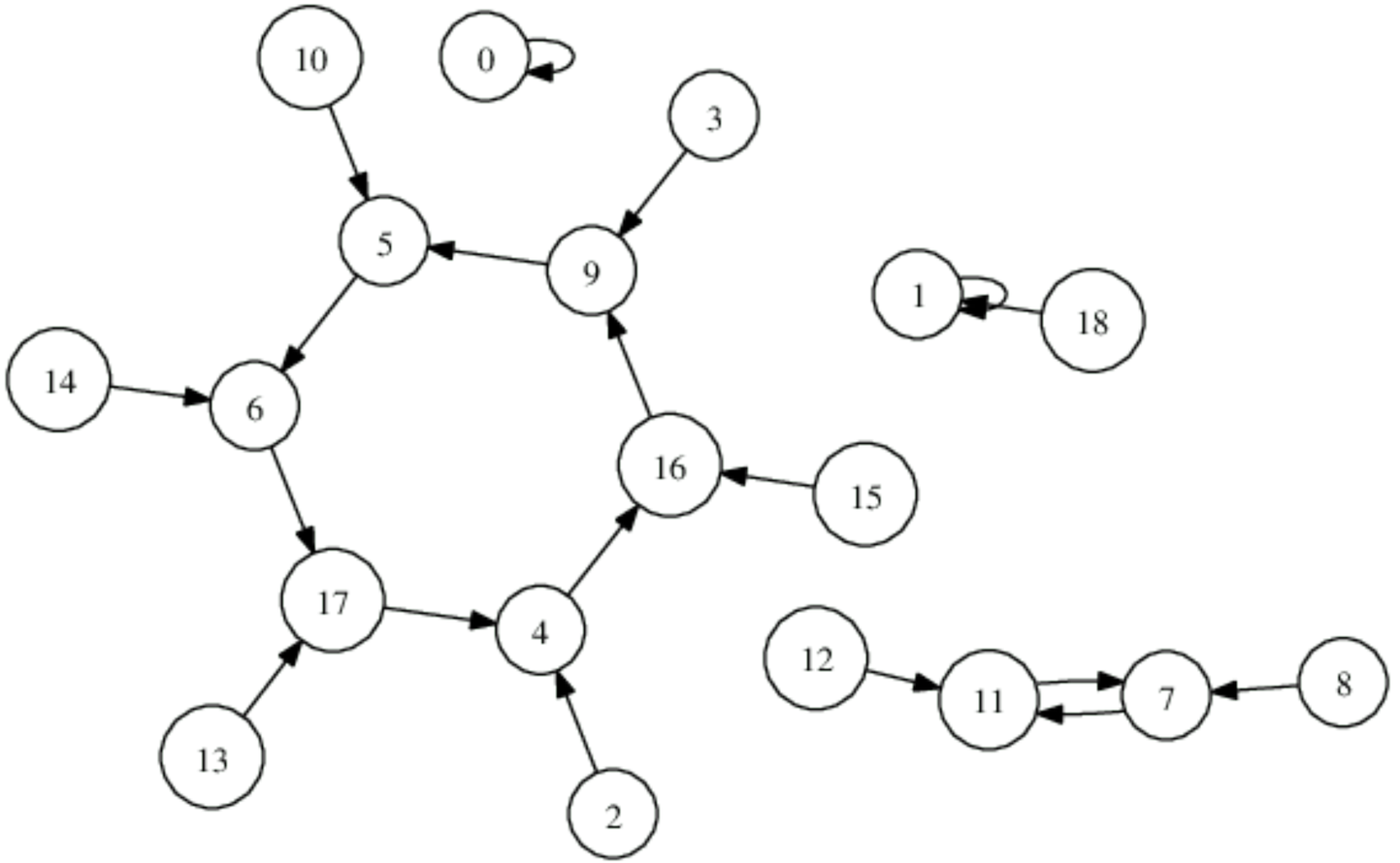

32
18
68
24
82
76
74
26
13
37
63
69
87
19
59
91
77
73
71
29
23
61
31
81
41
27
21
79
11
39
9
97
83
89
53
33
3
47
67
17
62
38
88
12
44
42
94
8
6
86
36
58
64
96
92
14
66
78
56
84
72
16
34
46
22
54
28
4
2
48
52
98
57
7
93
49
51
1
43
99
35
55
45
5
25
15
85
75
65
95
50
40
30
10
0
60
70
80
90
20



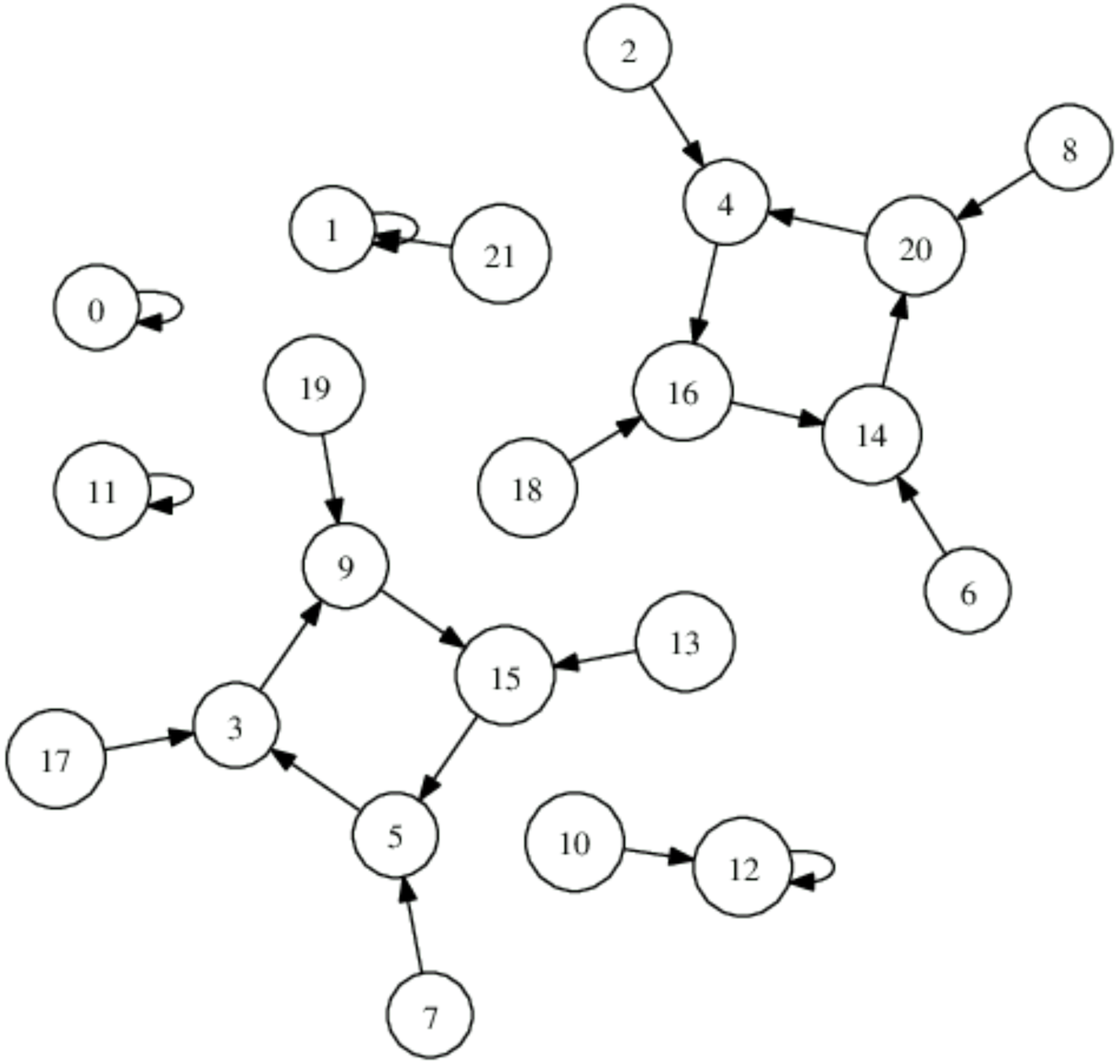
11 is has 4-cycle



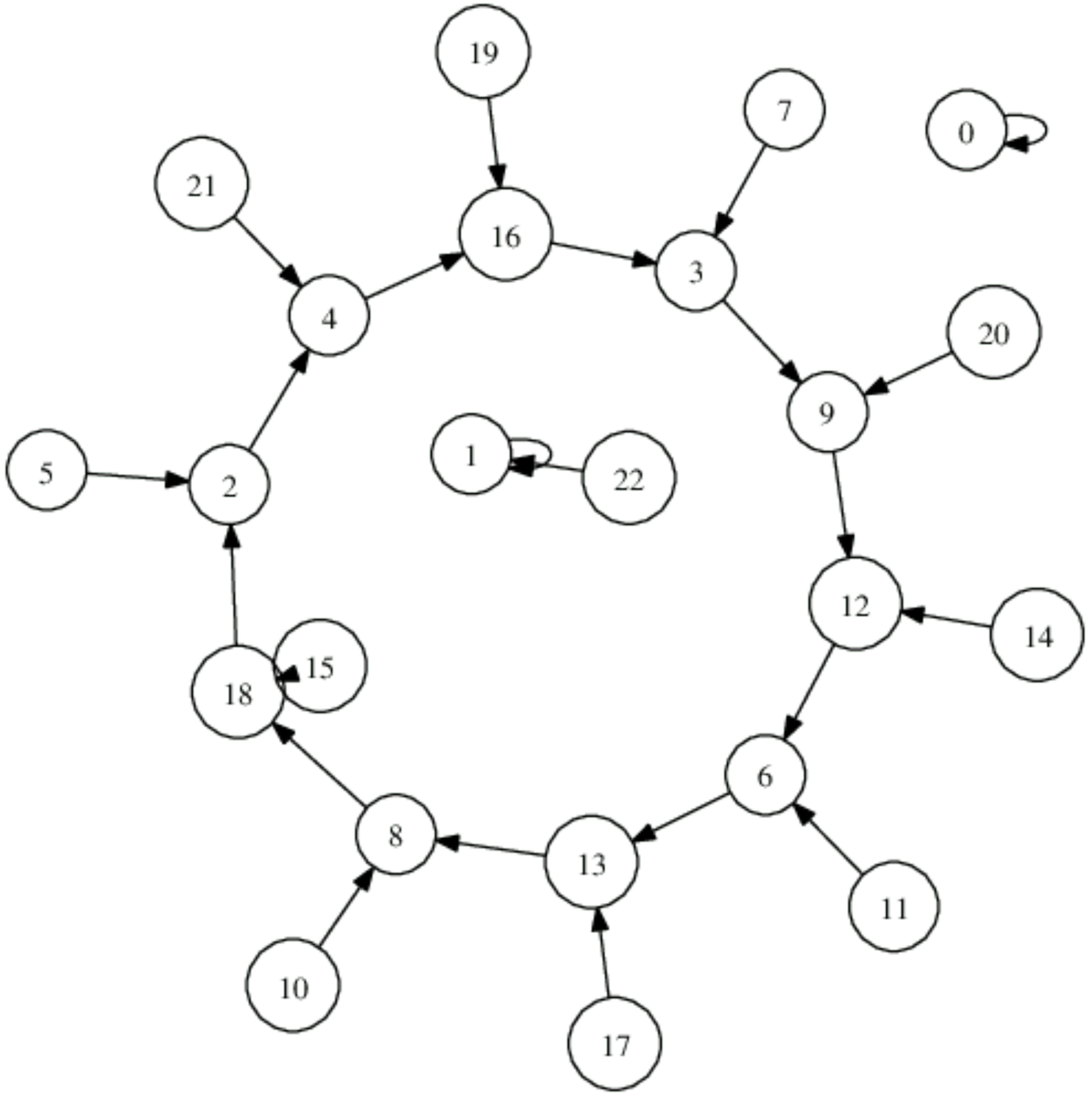
17 has a binary tree with 1 at the root, and 0 is a ‘looper’.



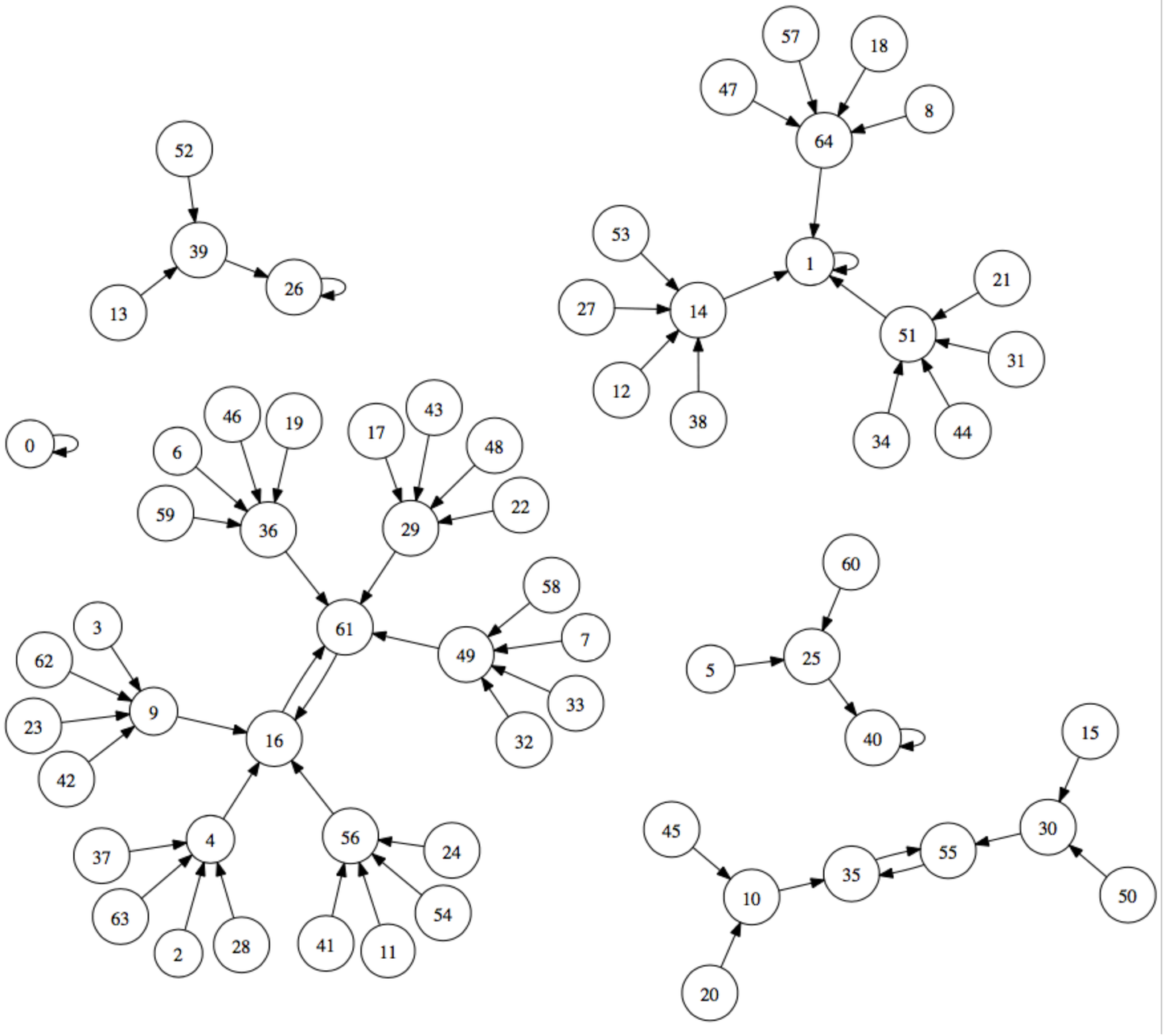
19 has a hexagon cycle. Note 11<->7 attractor…



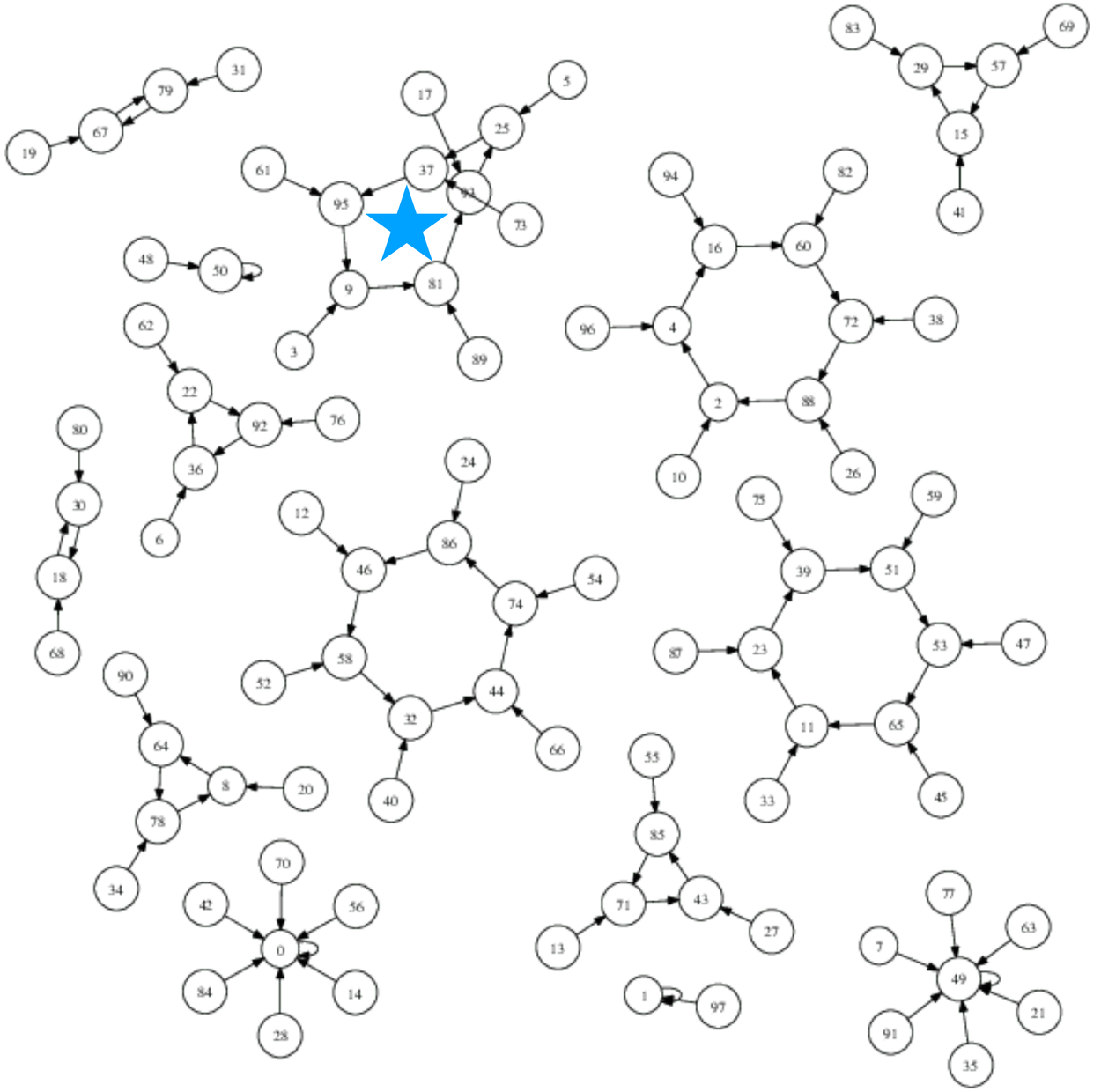
22 has TWO 4-cycles

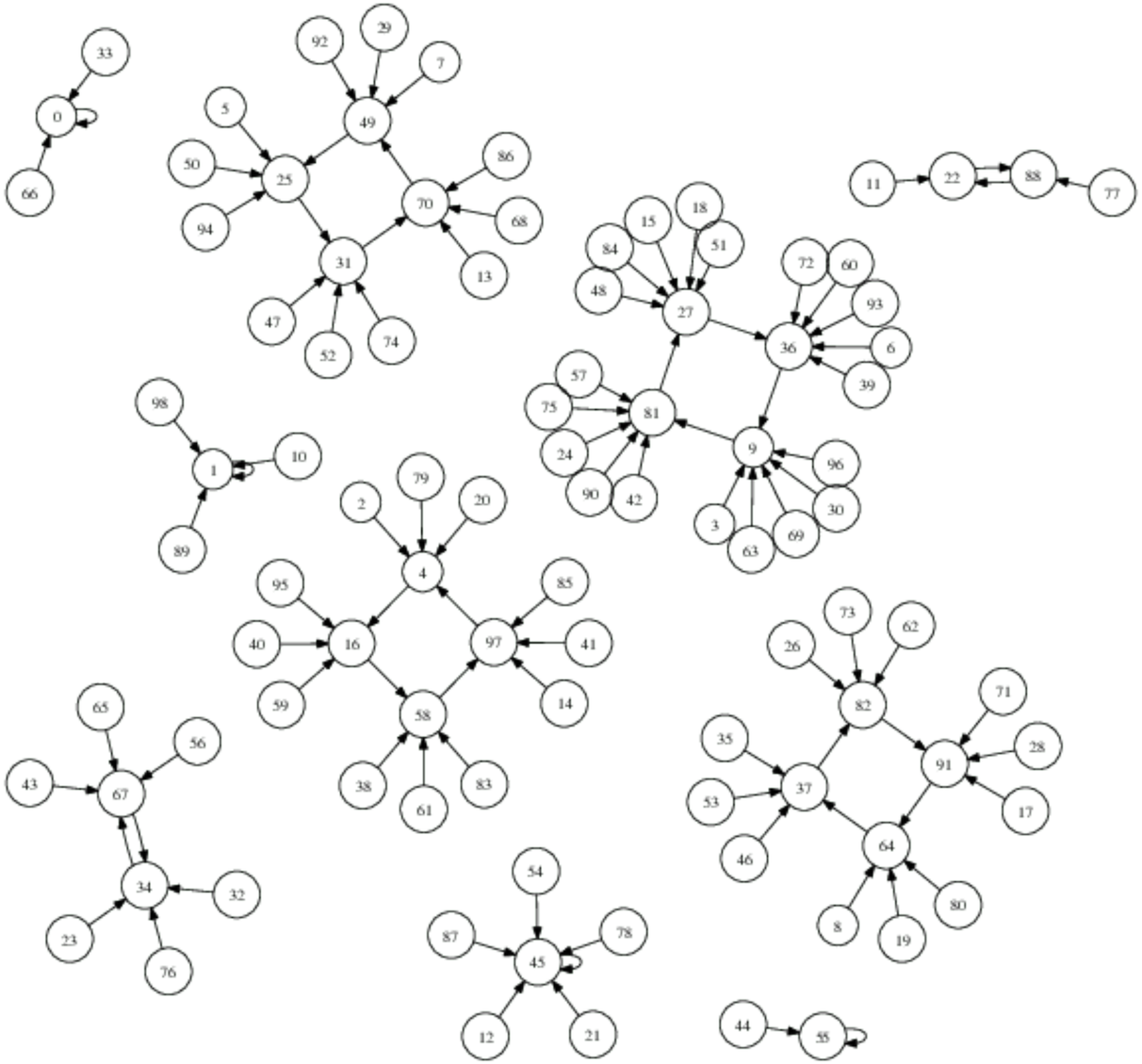


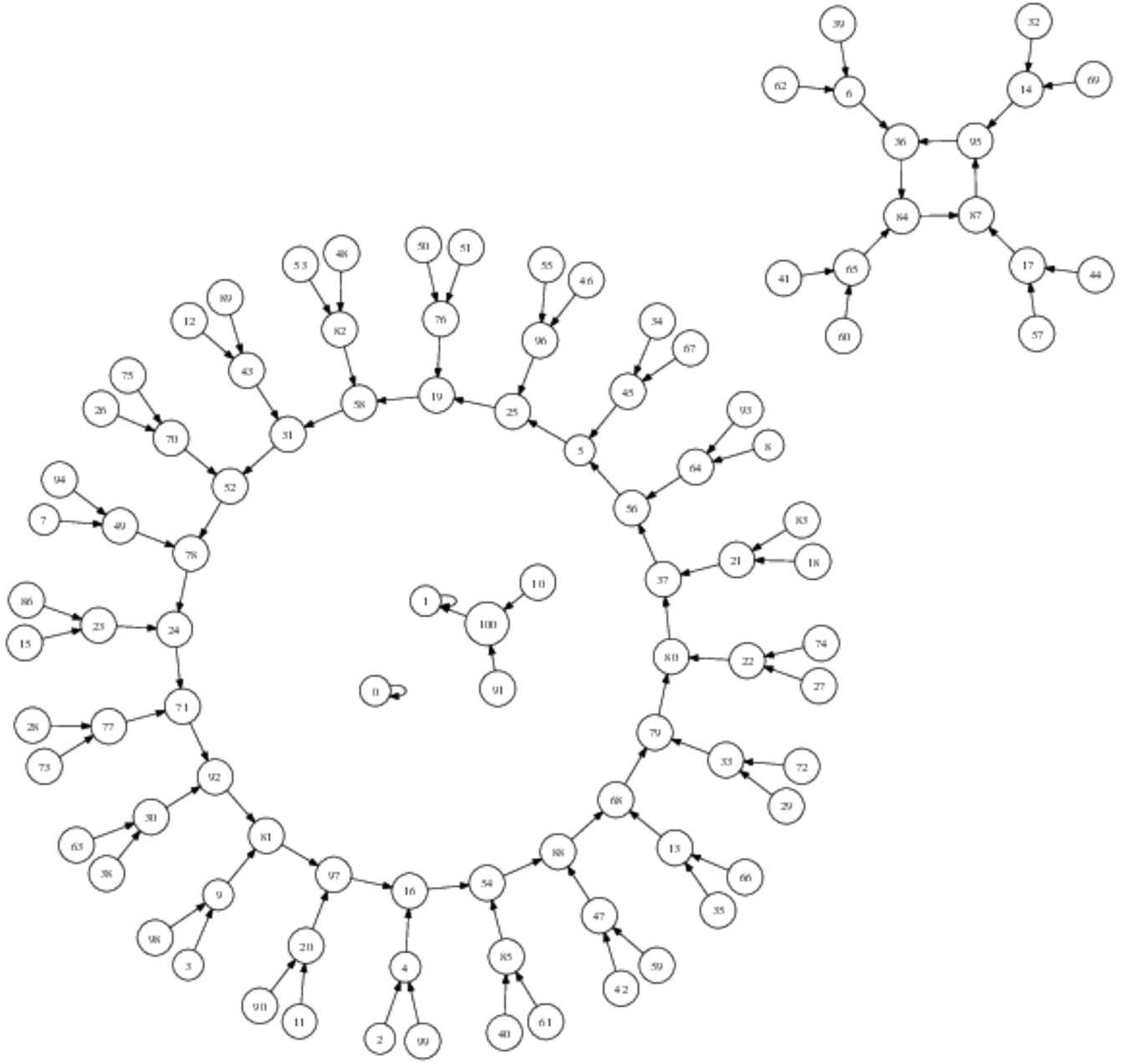
23 has a 10-cycle

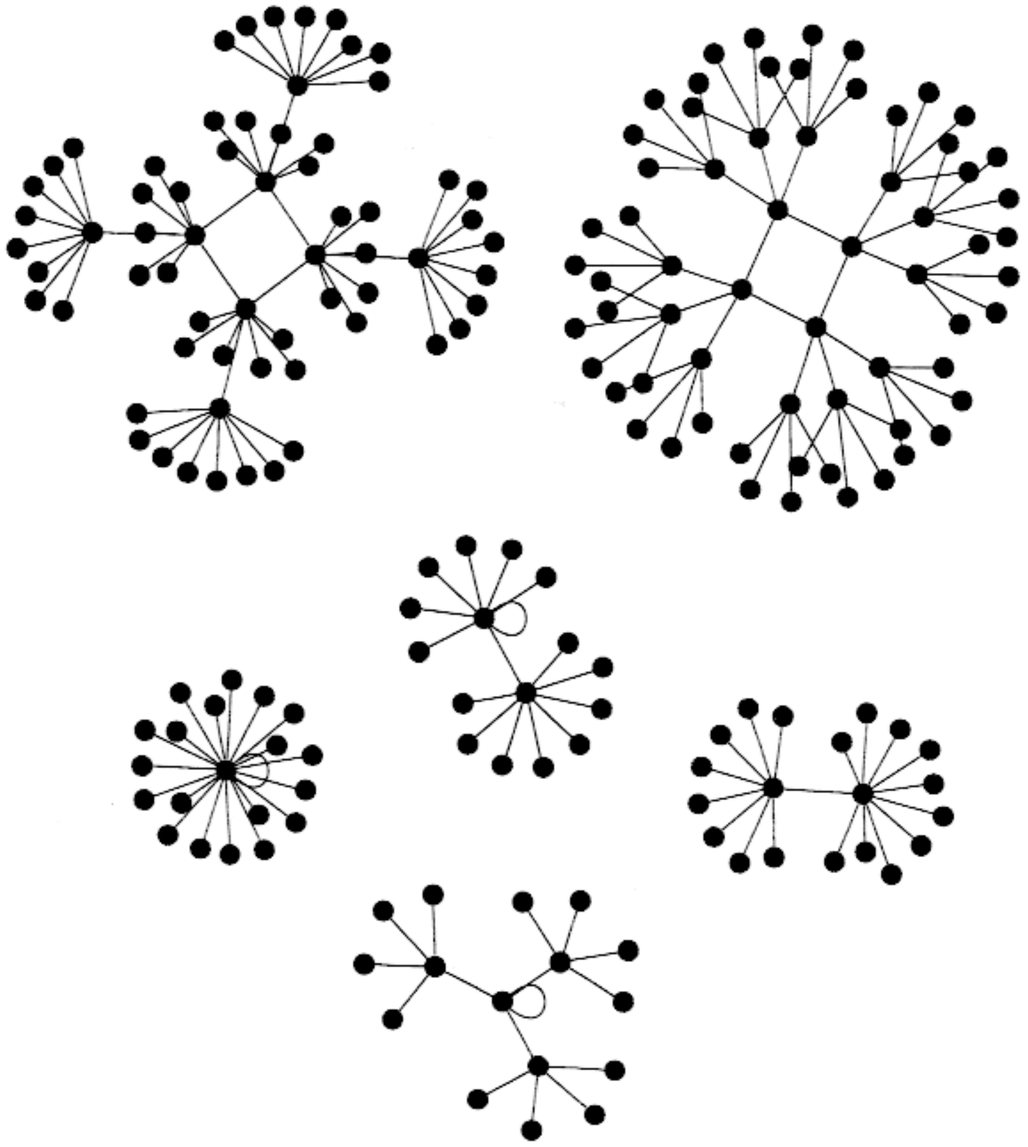


65 - An Attractor for your Birthday

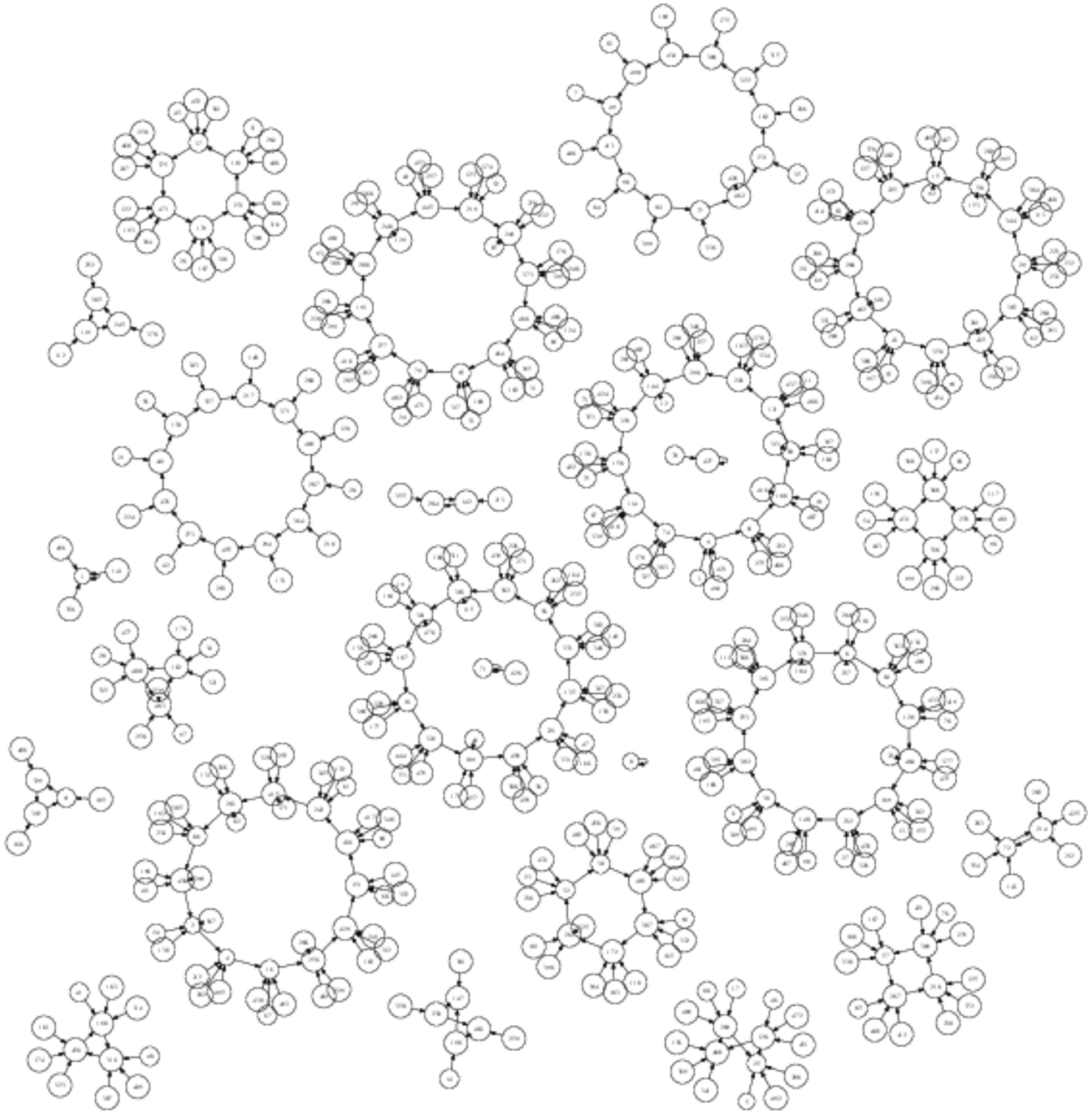

83
69
29
57
15
41
31
79
67
19
17
5
25
61
37
95
91
73
48
50
9
81
3
89
94
82
16
60
96
4
72
38
2
88
10
26
62
22
92
76
36
6
80
30
18
68
24
12
86
46
54
74
58
52
44
32
66
40
90
64
8
20
78
34
70
42
56
0
84
14
28
75
59
39
51
87
23
53
47
11
65
33
45
55
85
71
43
13
27
1
97
77
7
63
49
91
21
35

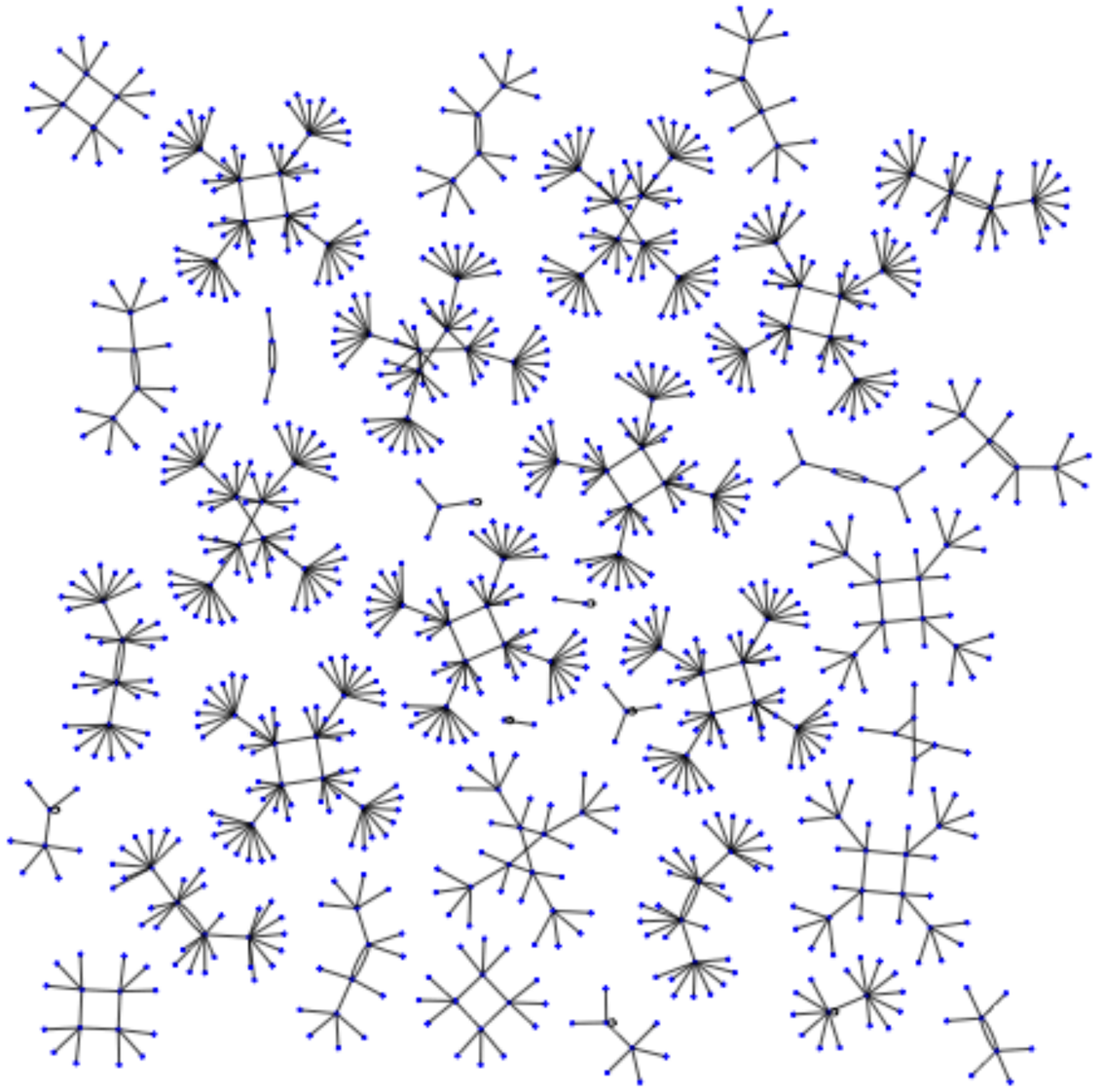

33
0
66
92
29
7
5
49
50
25
86
70
94
68
31
13
47
52
74
11
22
88
77
15
18
84
51
72
60
48
27
93
36
6
57
39
75
81
24
9
96
90
42
30
3
69
63
98
1
10
89
2
79
20
4
95
85
40
16
97
41
59
14
58
38
61
83
65
56
43
67
34
32
23
76
54
87
78
45
12
21
73
62
26
82
71
35
91
28
53
37
17
46
64
8
19
80
44
55



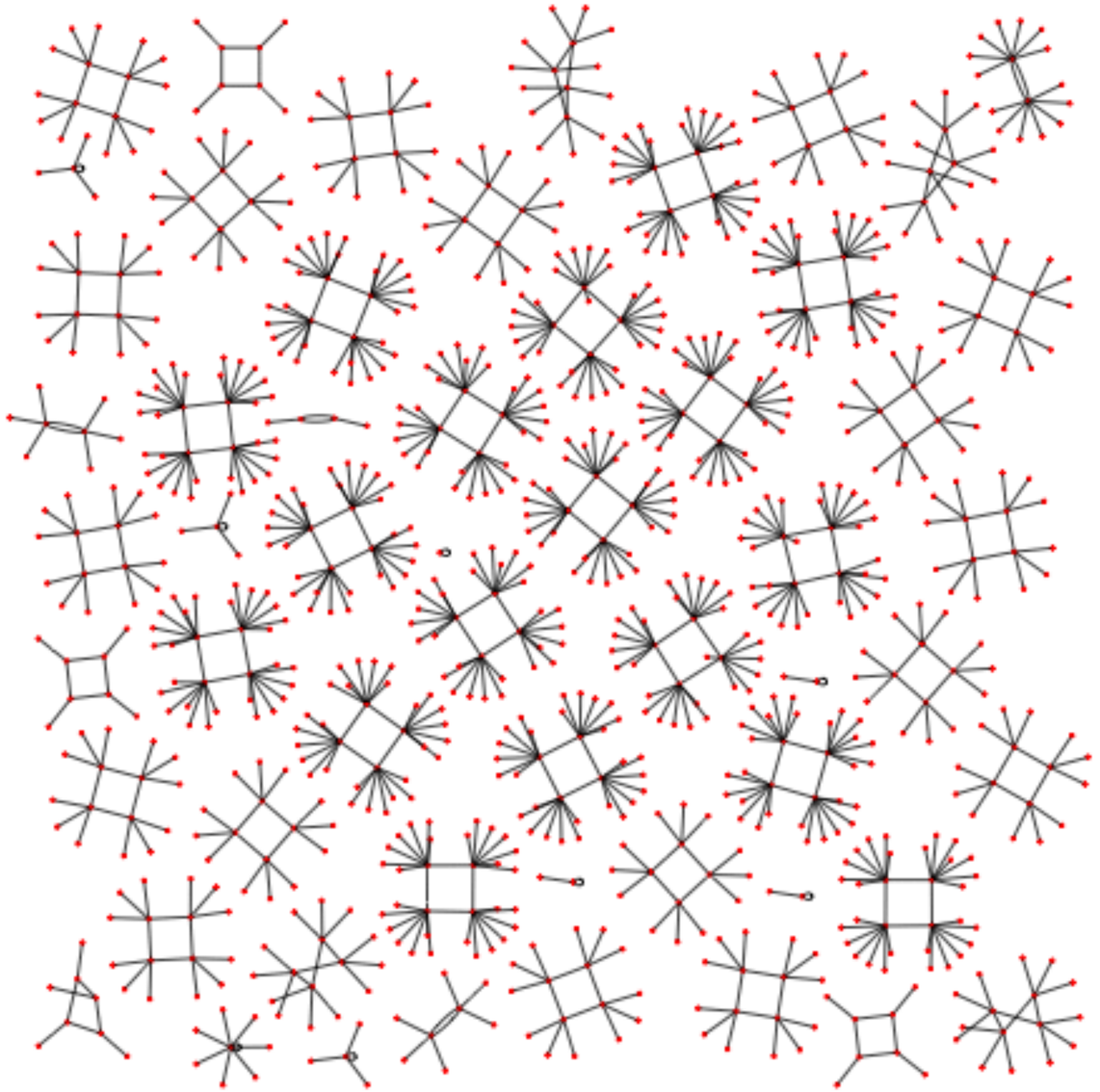


200





1001



1023

I have 500+ of these,

which I plan to make available,

as a web page and PDF (G4G15?)

Collatz sequence

n -> (3n+1,n/2)

if ($x % 2 == 0) { return $x / 2; } else { return 3 * $x + 1; }

This defines a unique successor for every integer!

Examples:

22 -> 22/2 = 11

21 -> 3 * 21 + 1 = 64

2 -> 1; 3 -> 10; 10 -> 5; 5 -> 16; 16 -> 8;

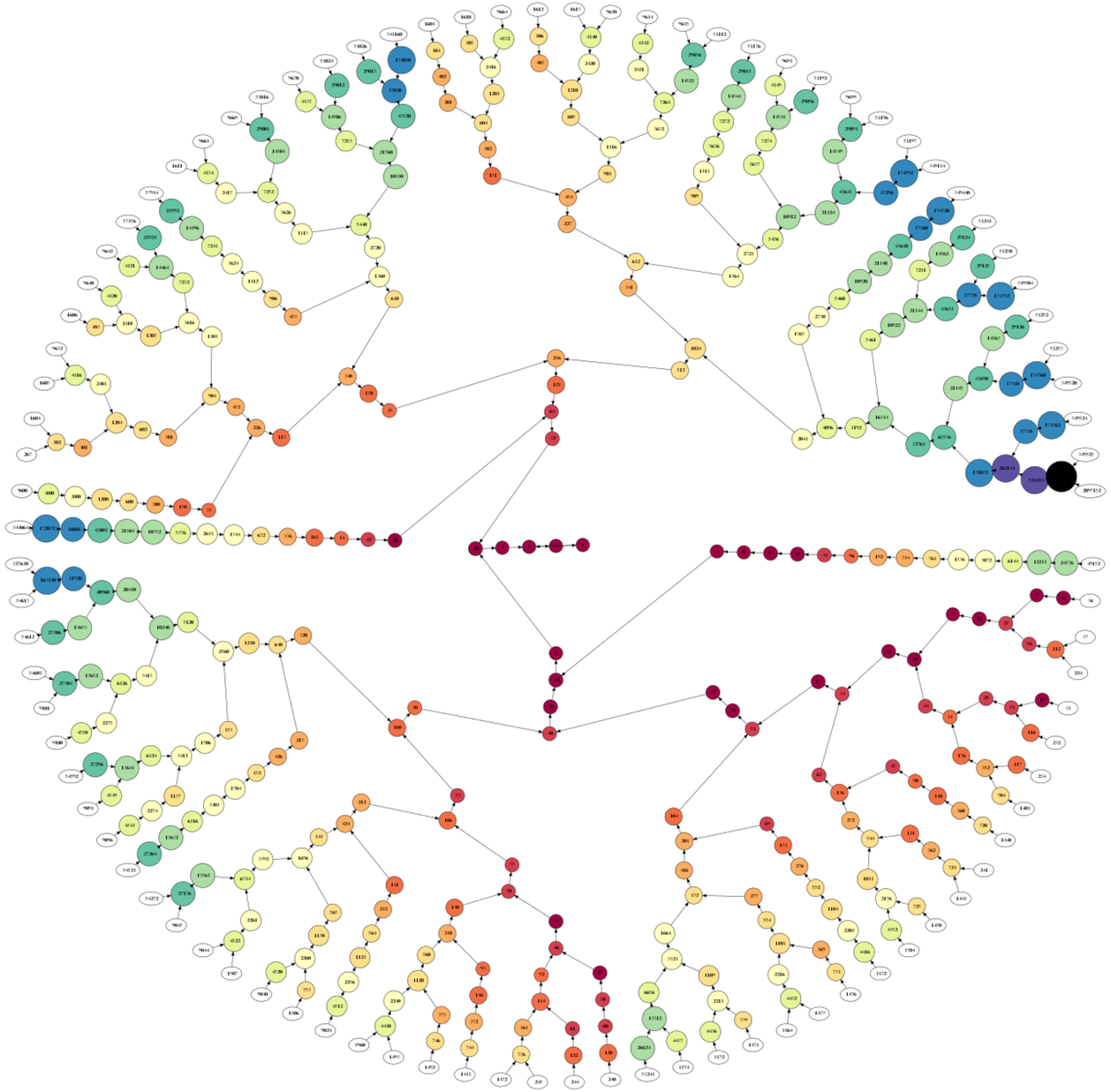
8 -> 4; 4 -> 2; 6 -> 3; 7 -> 22; 22 -> 11;

11 -> 34; 34 -> 17; 17 -> 52; 52 -> 26;

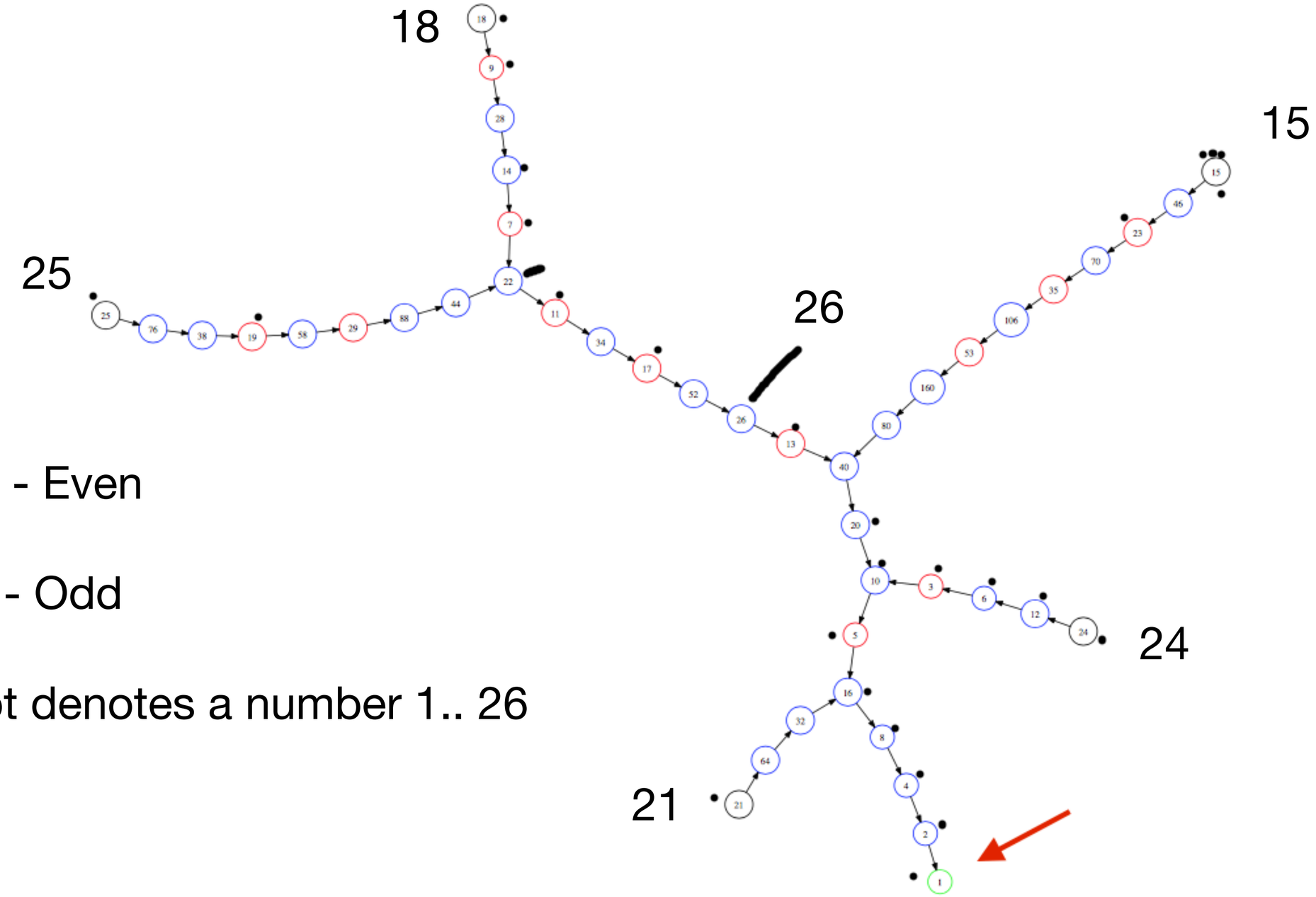
26 -> 13; 13 -> 40; 40 -> 20; 20 -> 10;

9 -> 28; 28 -> 14; 14 -> 7; 12 -> 6; 15 -> 46;

Wikipedia



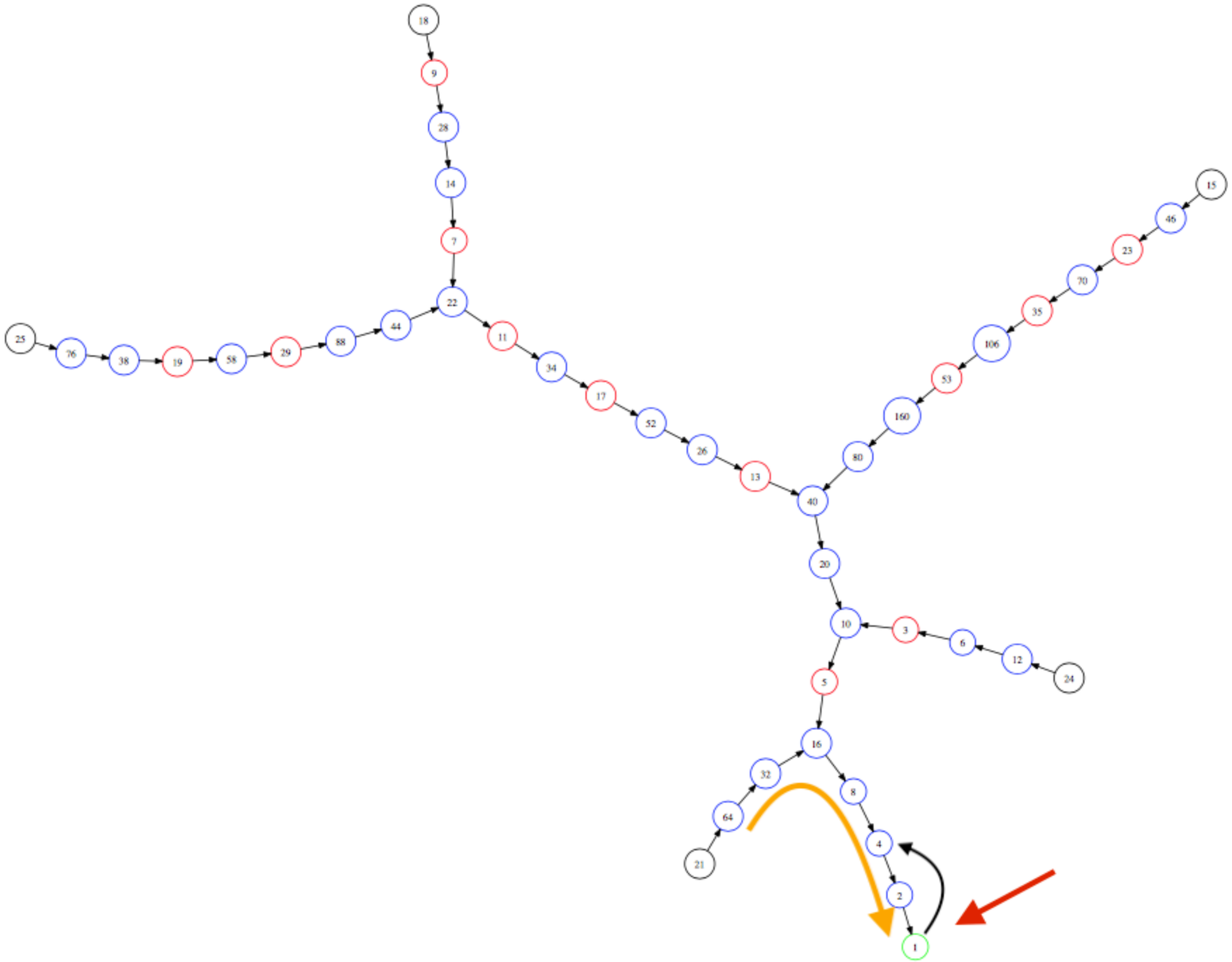
All Collatz sequences of a length inferior to 20



Blue - Even

Red - Odd

A dot denotes a number 1.. 26

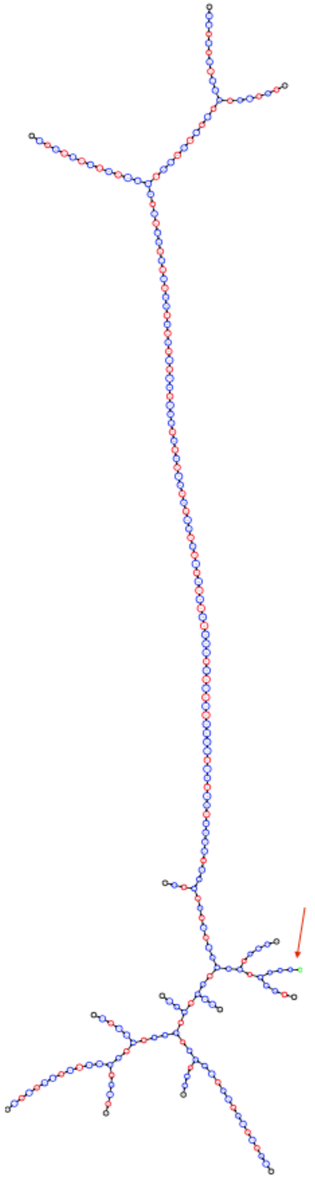


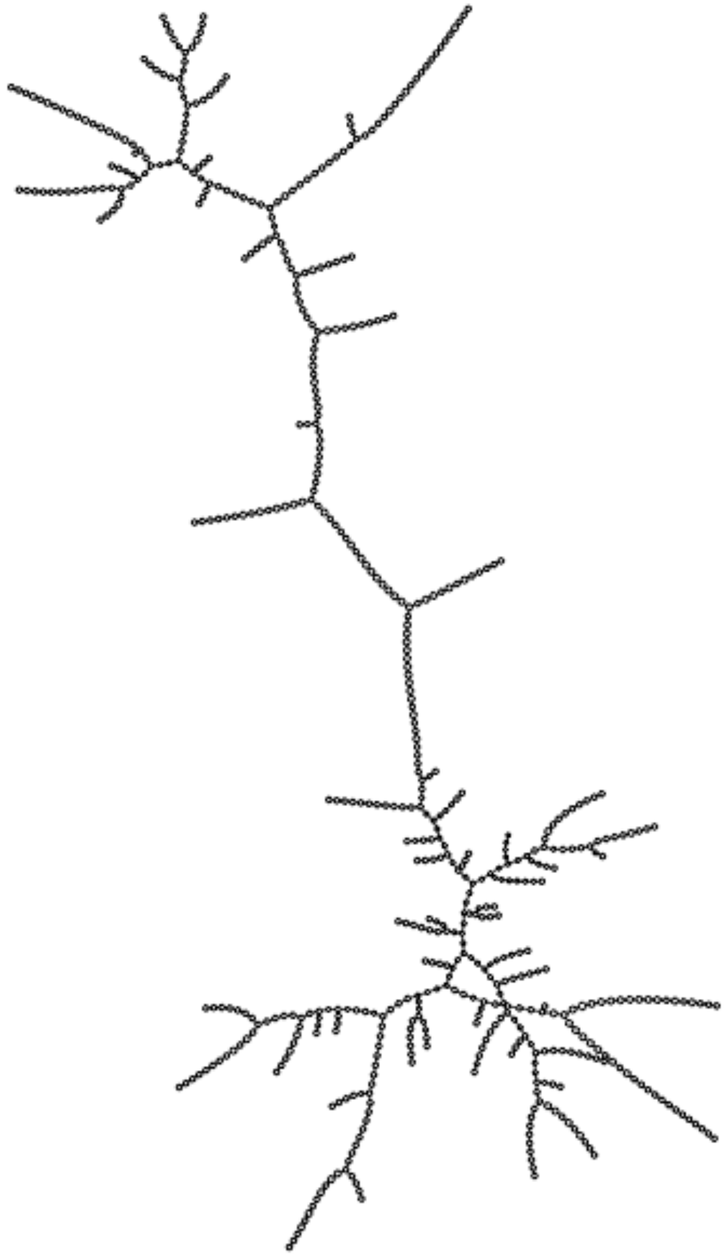
(26) See the little 4-2-1 loop at the end.

EXERCISE:

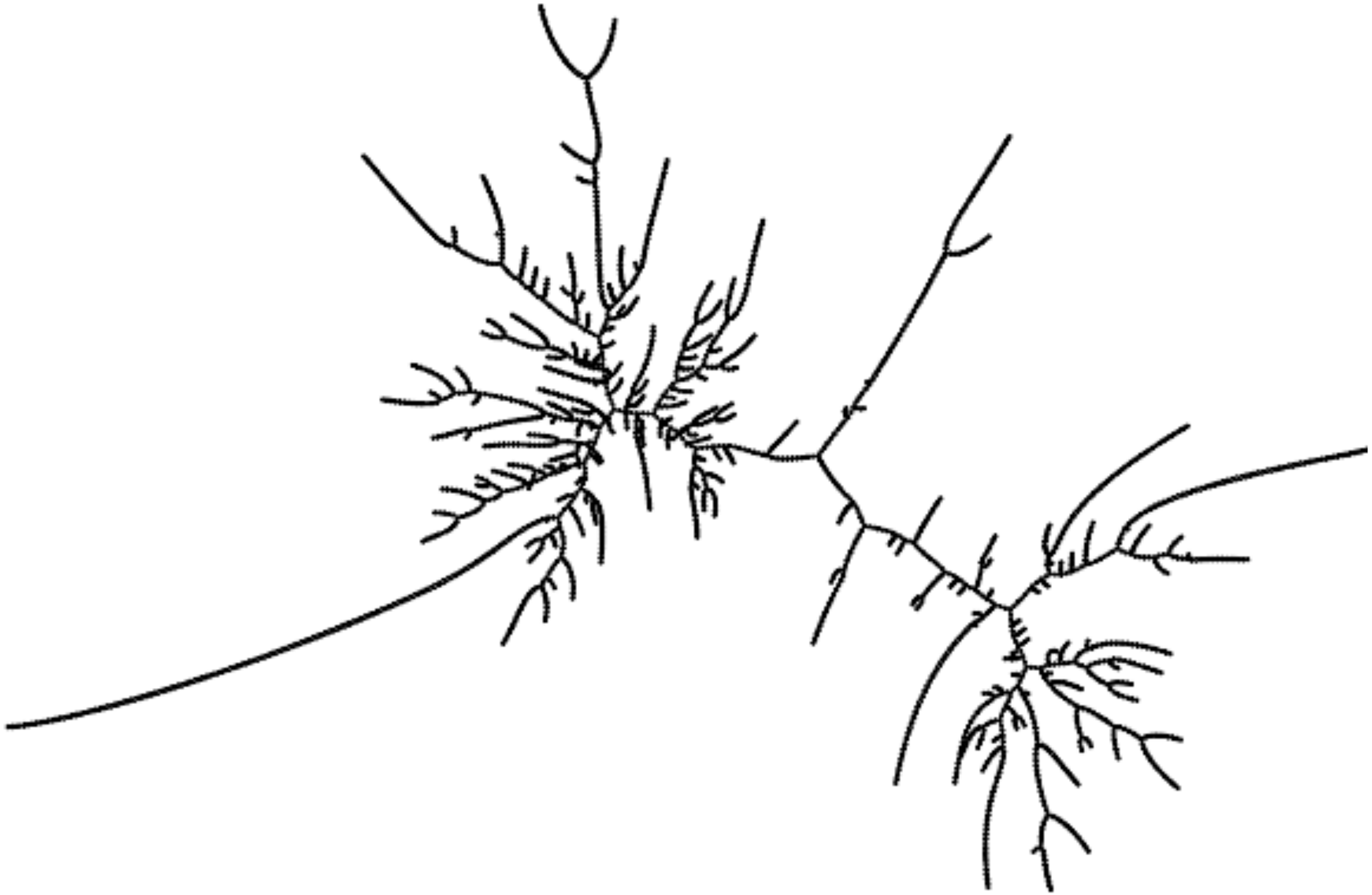
Compute the Collatz Sequence starting with 27.

(JHC)





300



1000

Conjecture

All integers are in one CONNECTED graph.

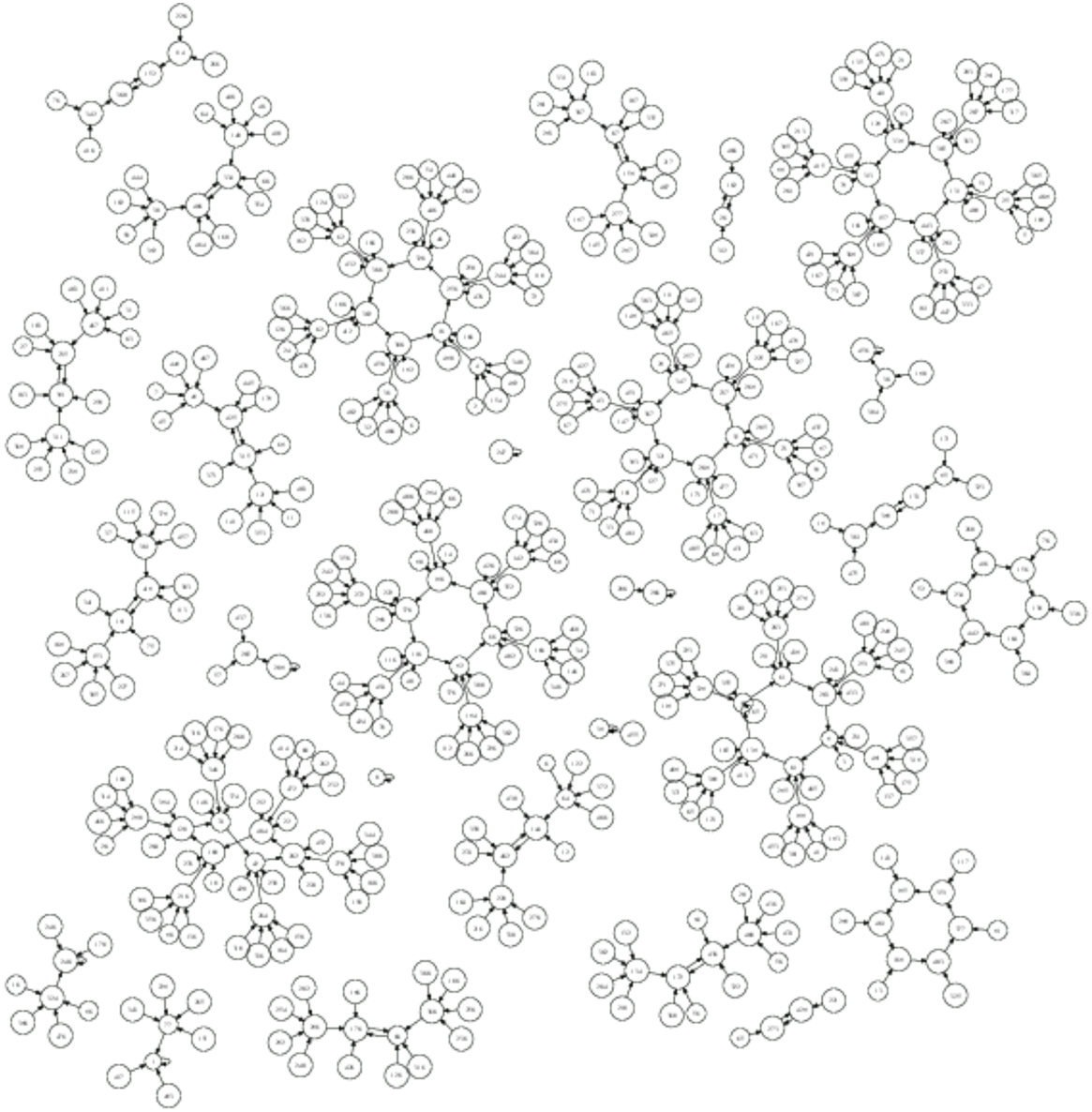
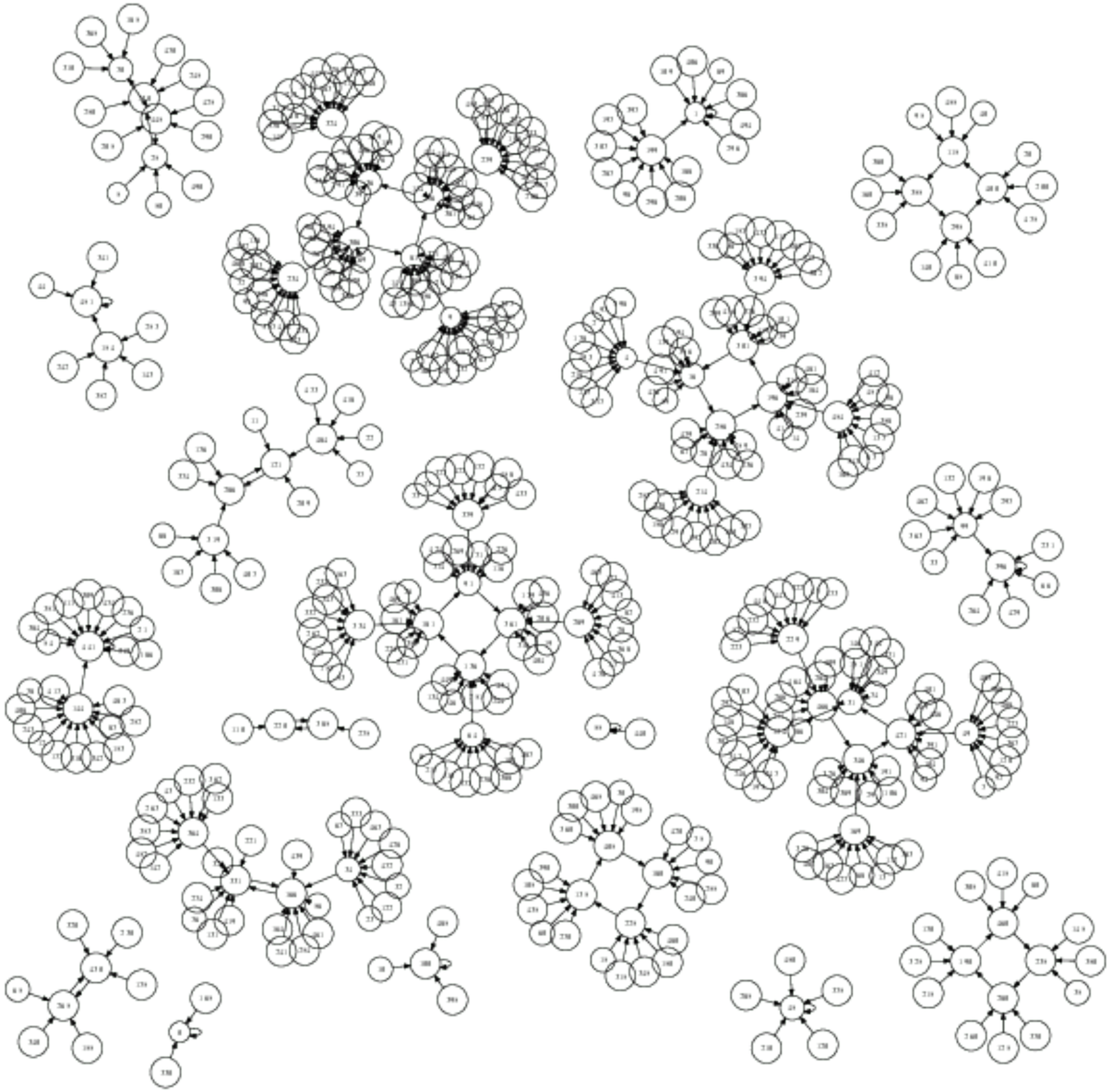
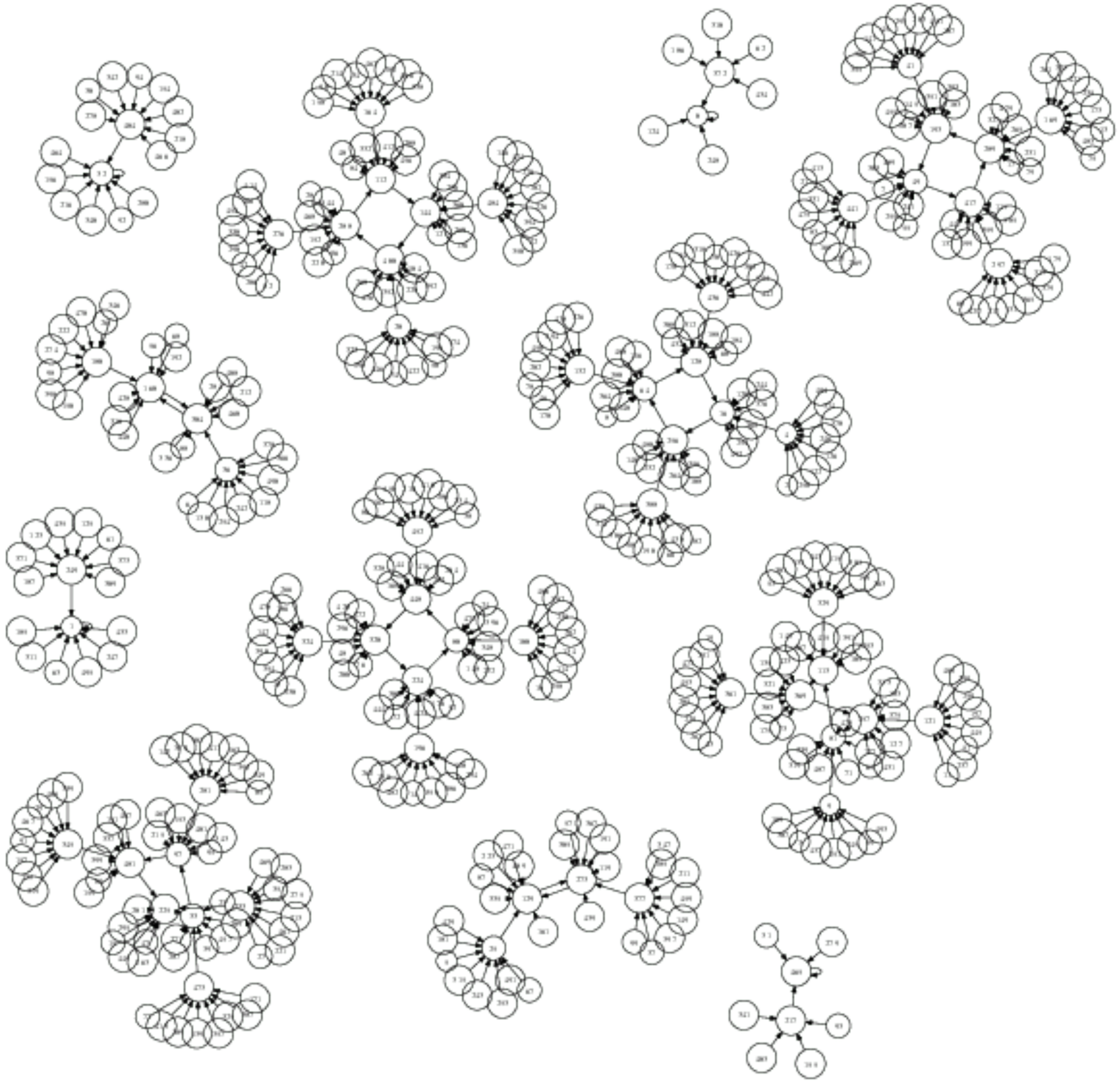
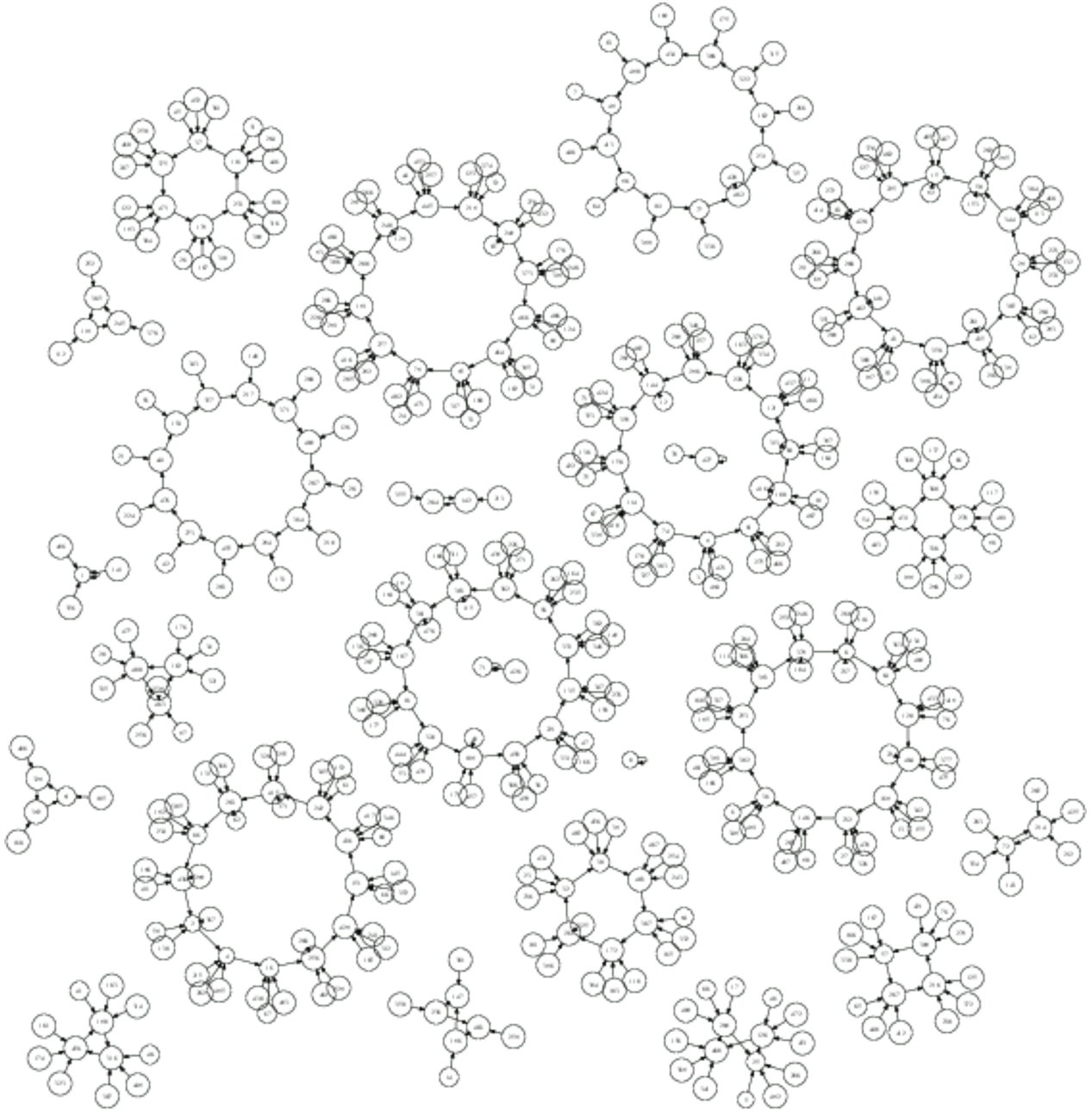
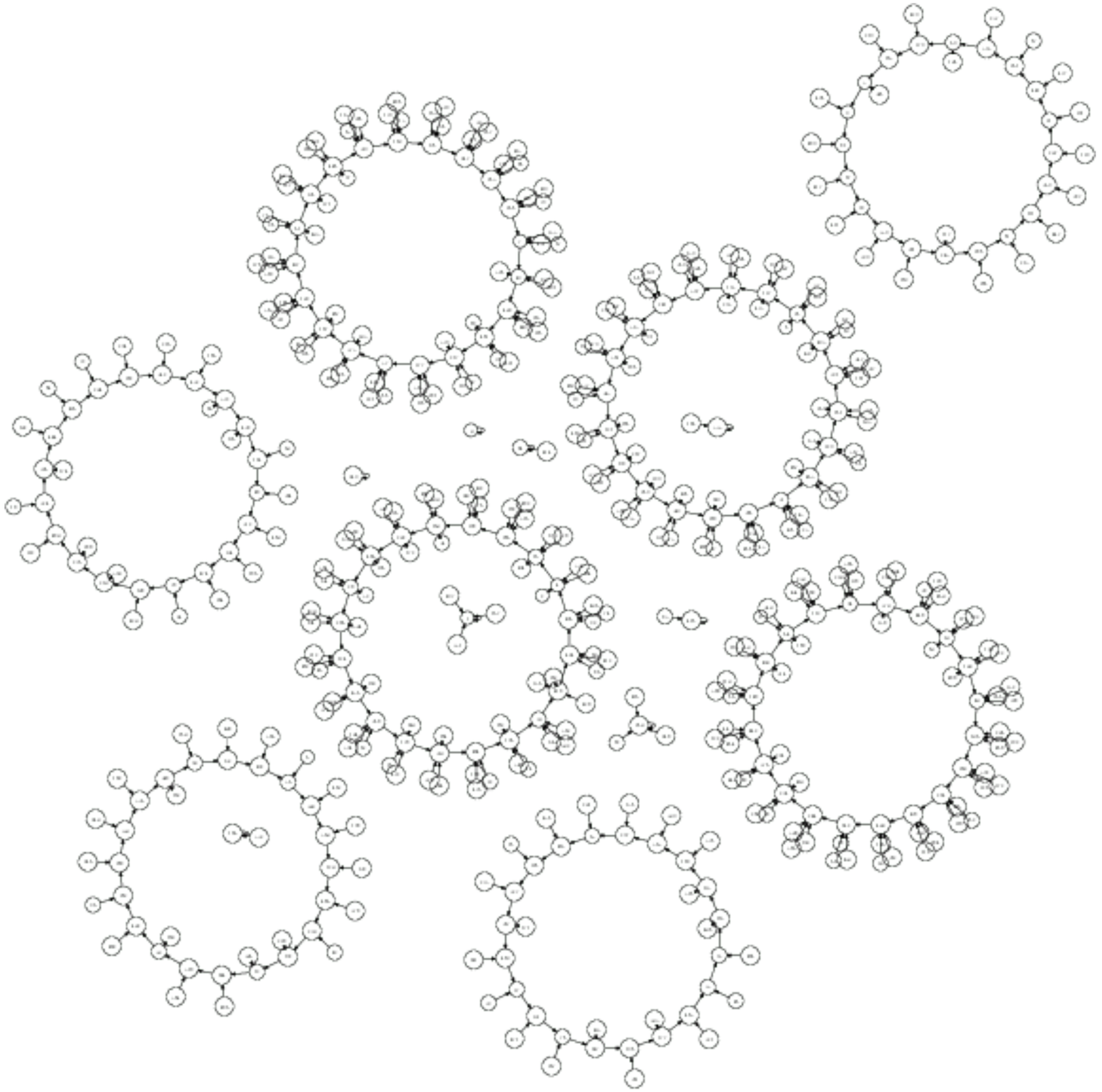
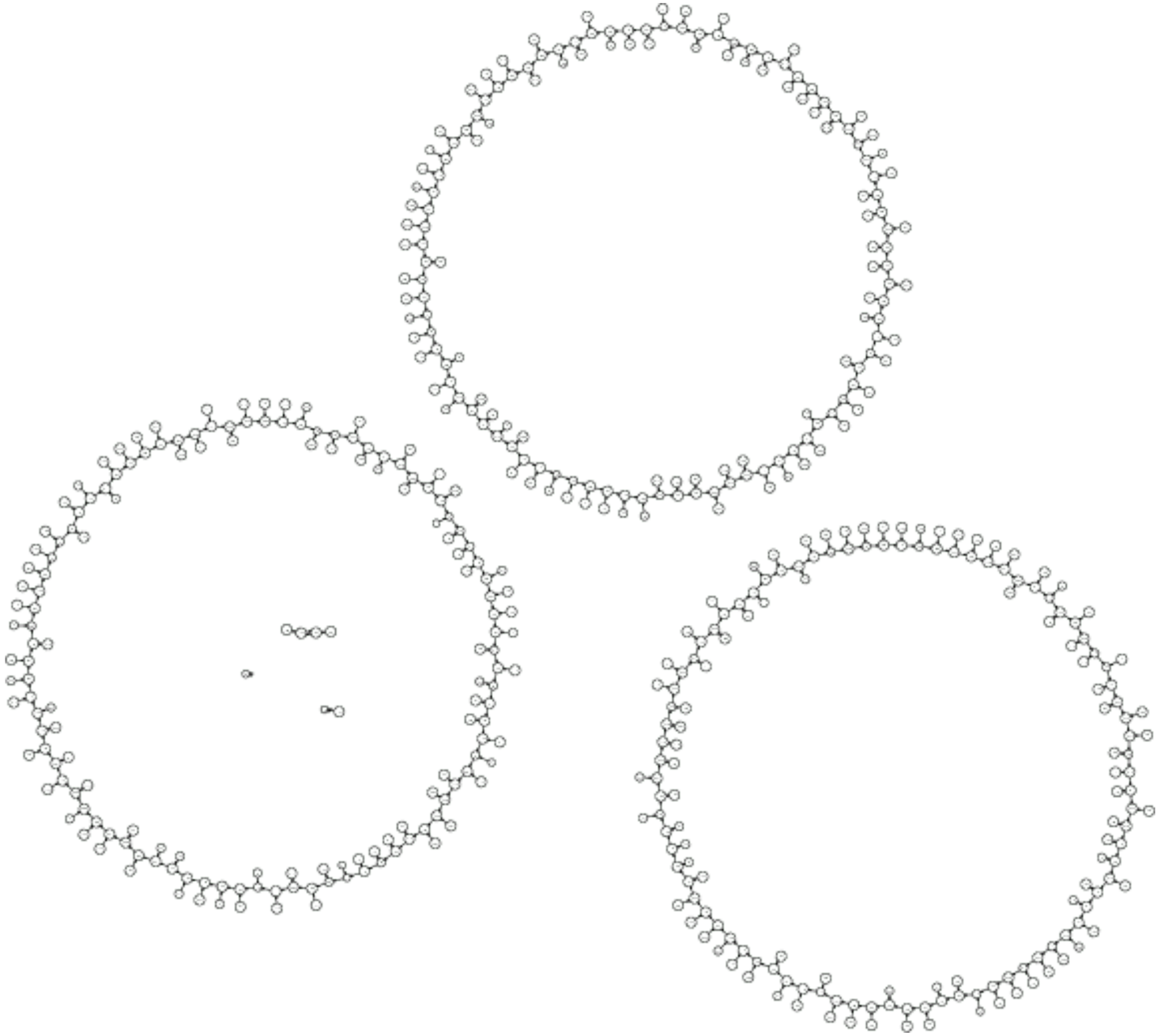
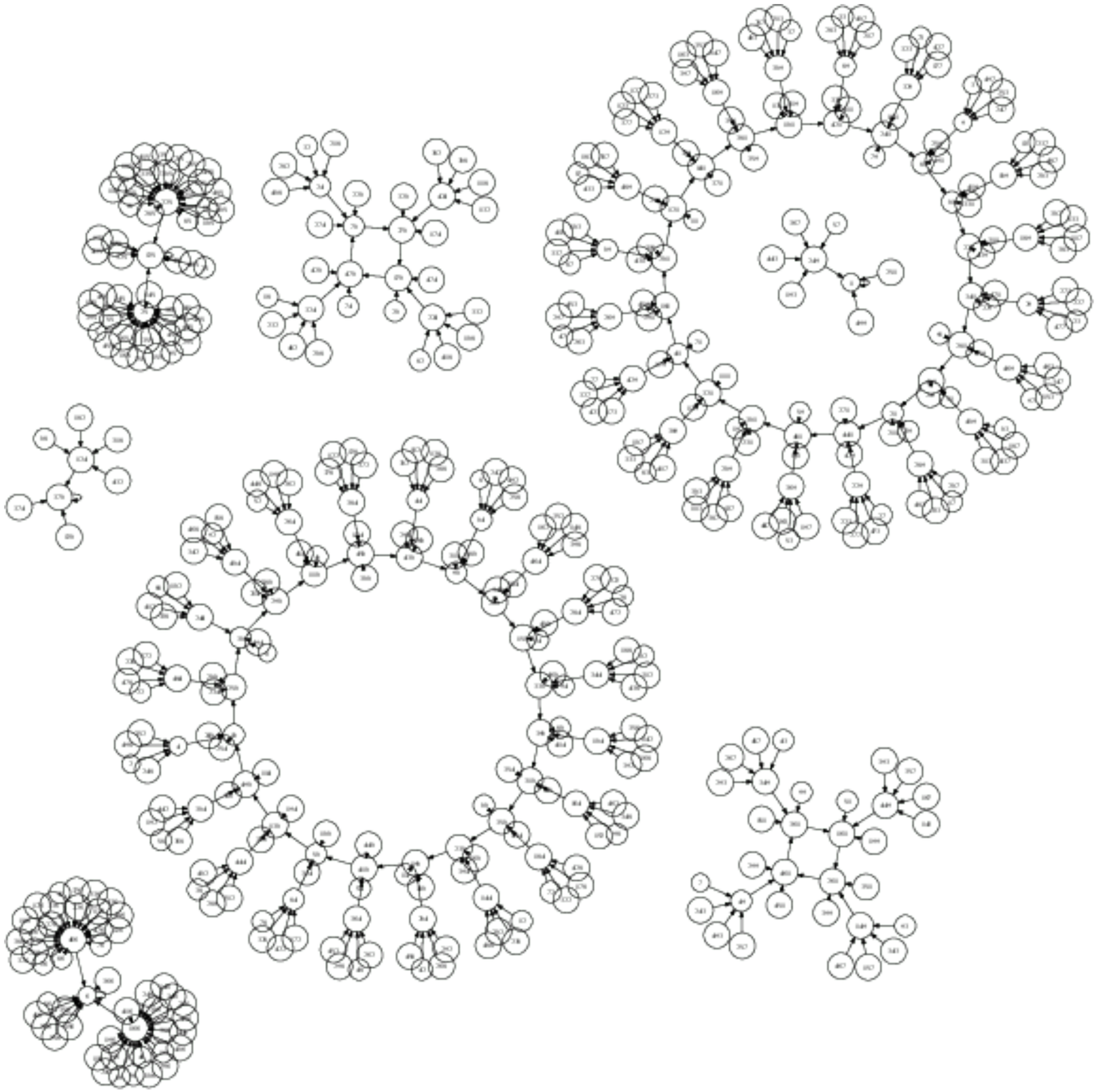
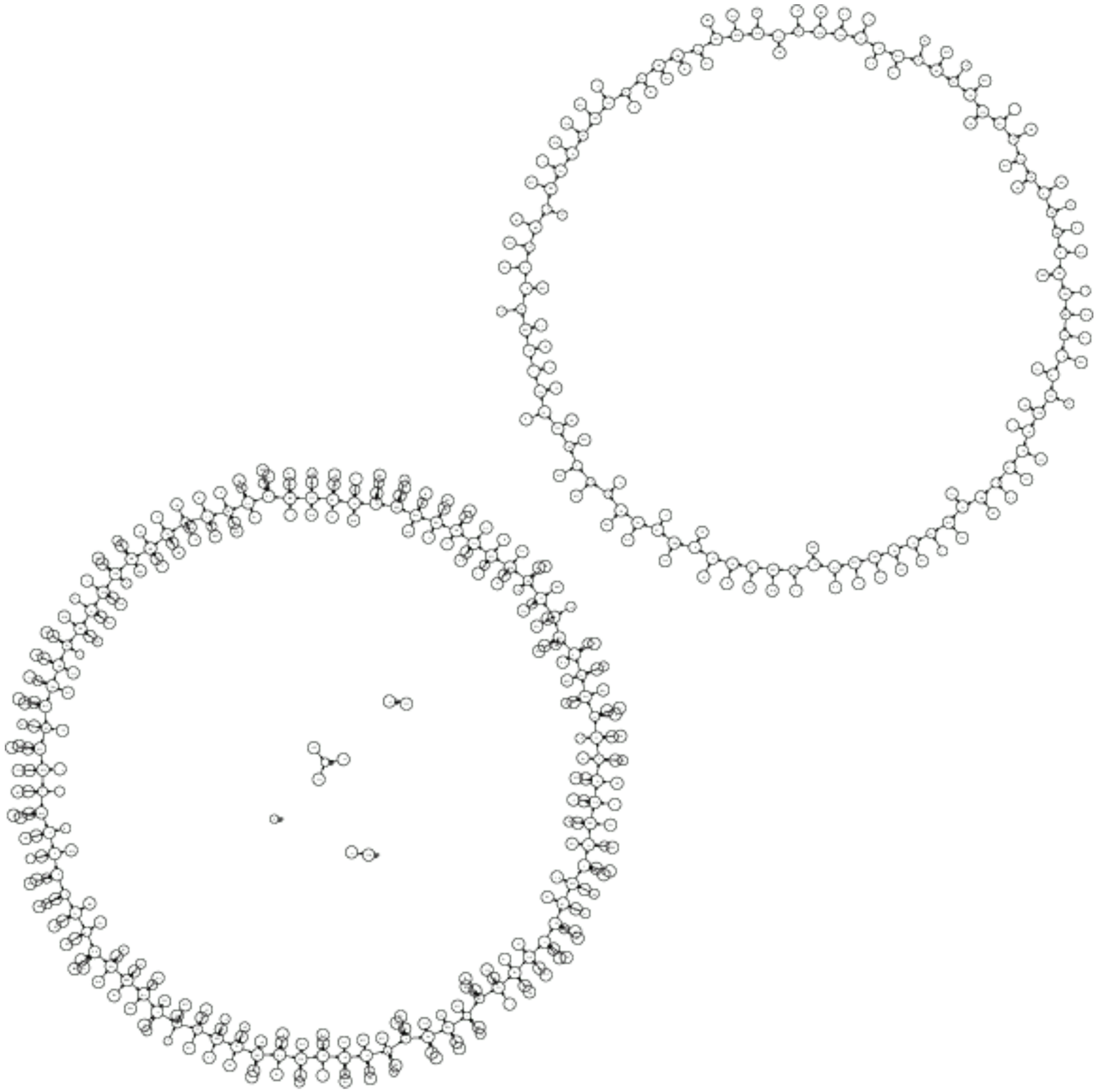
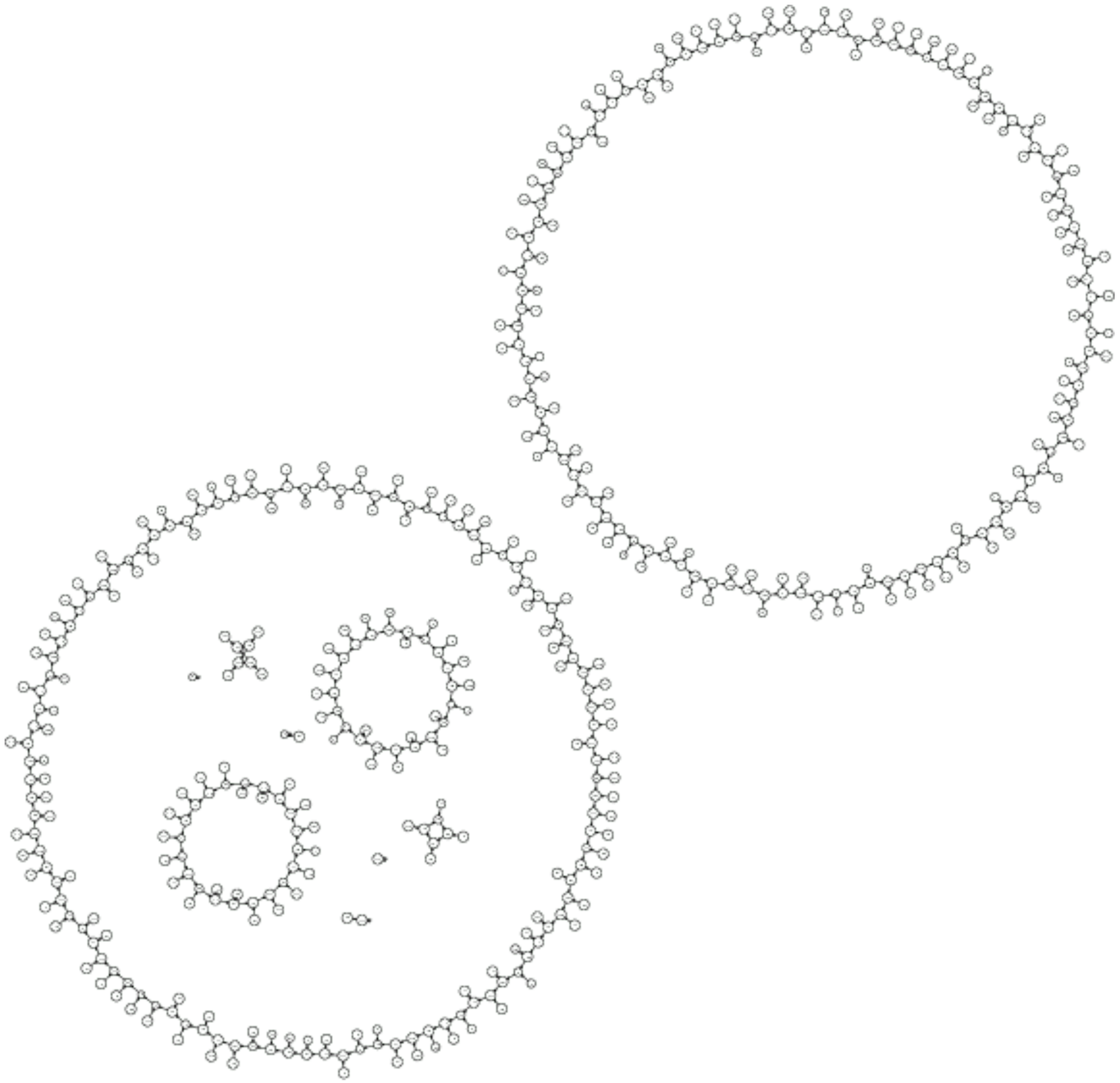
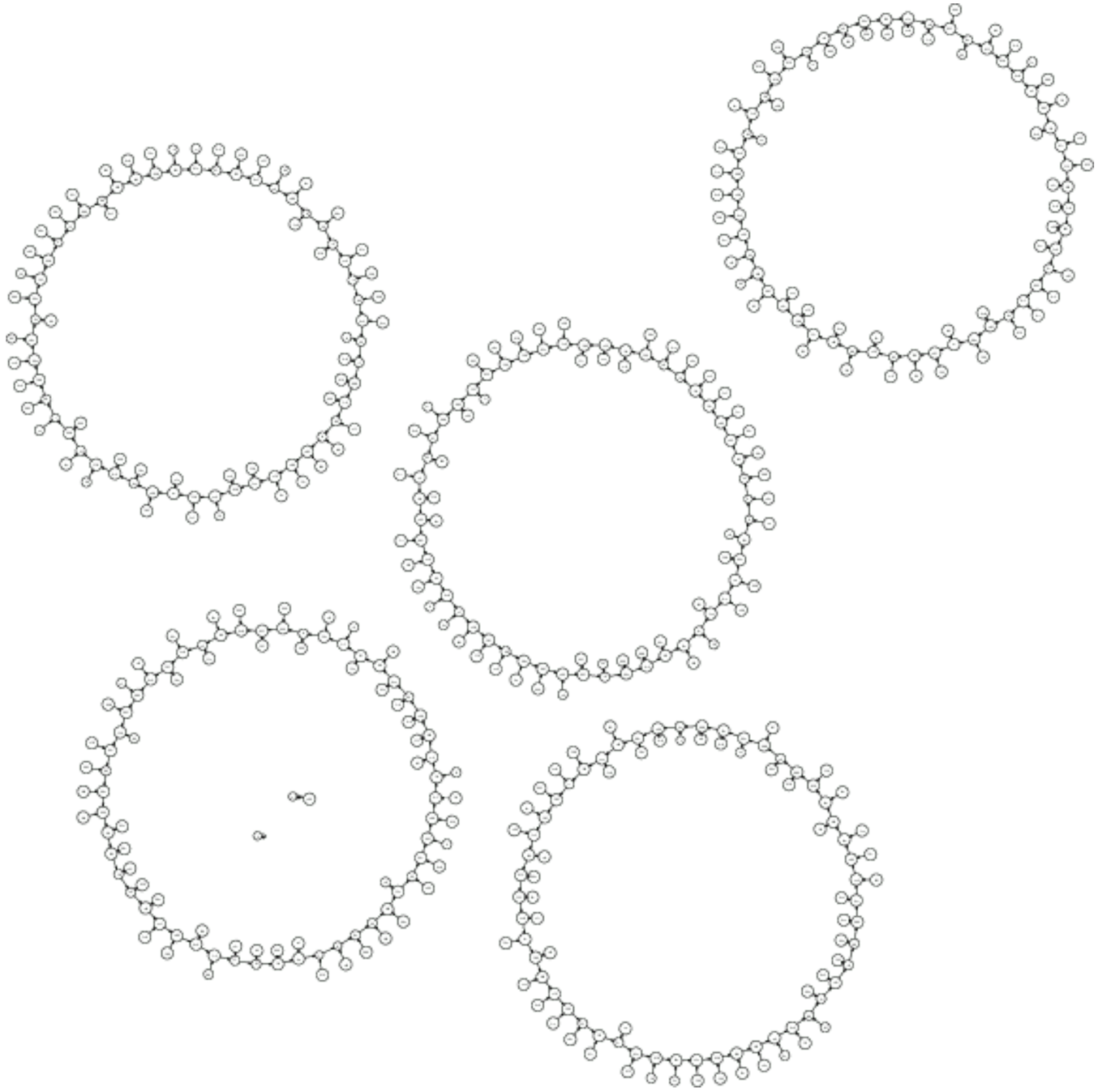
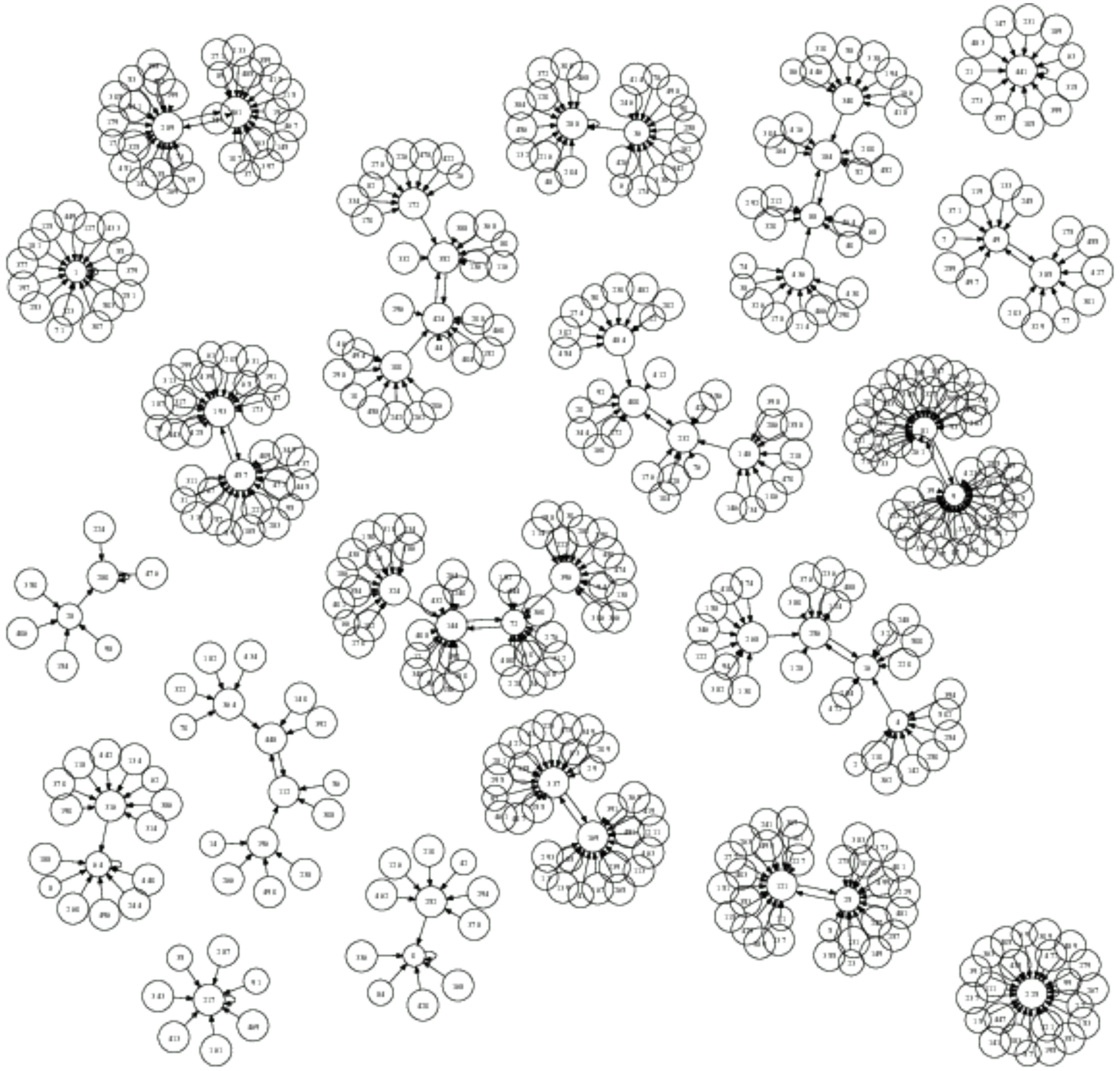
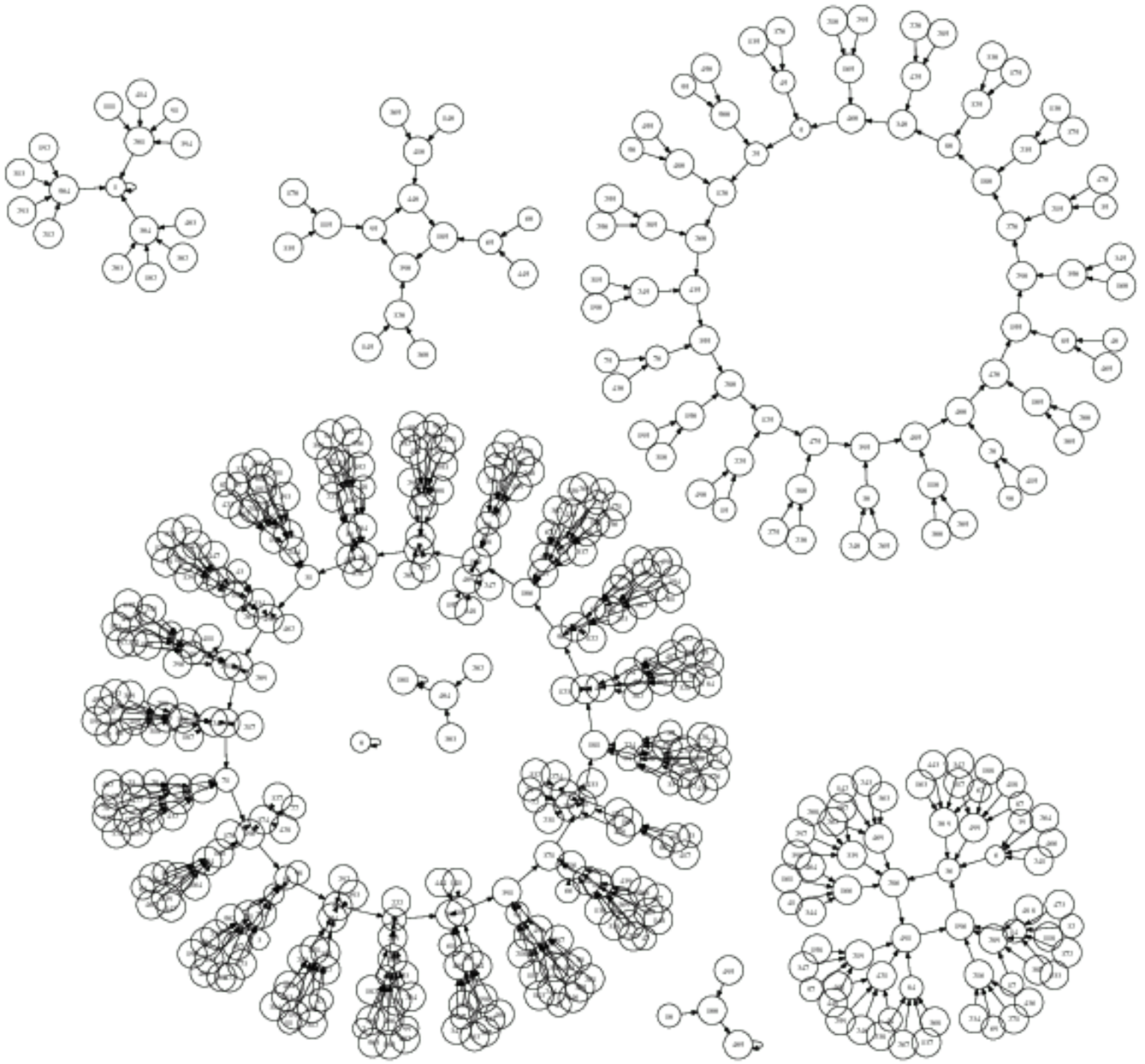
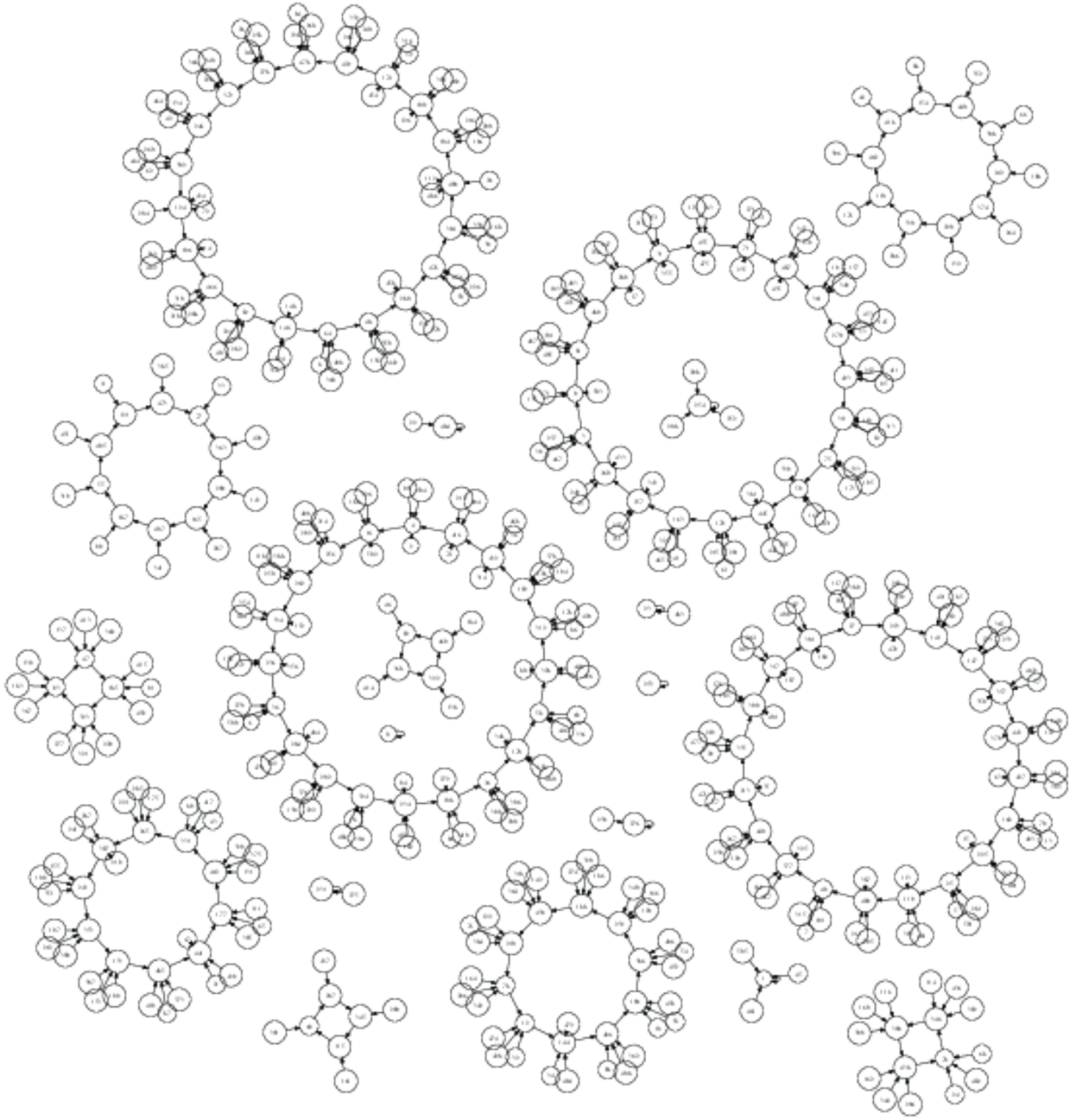
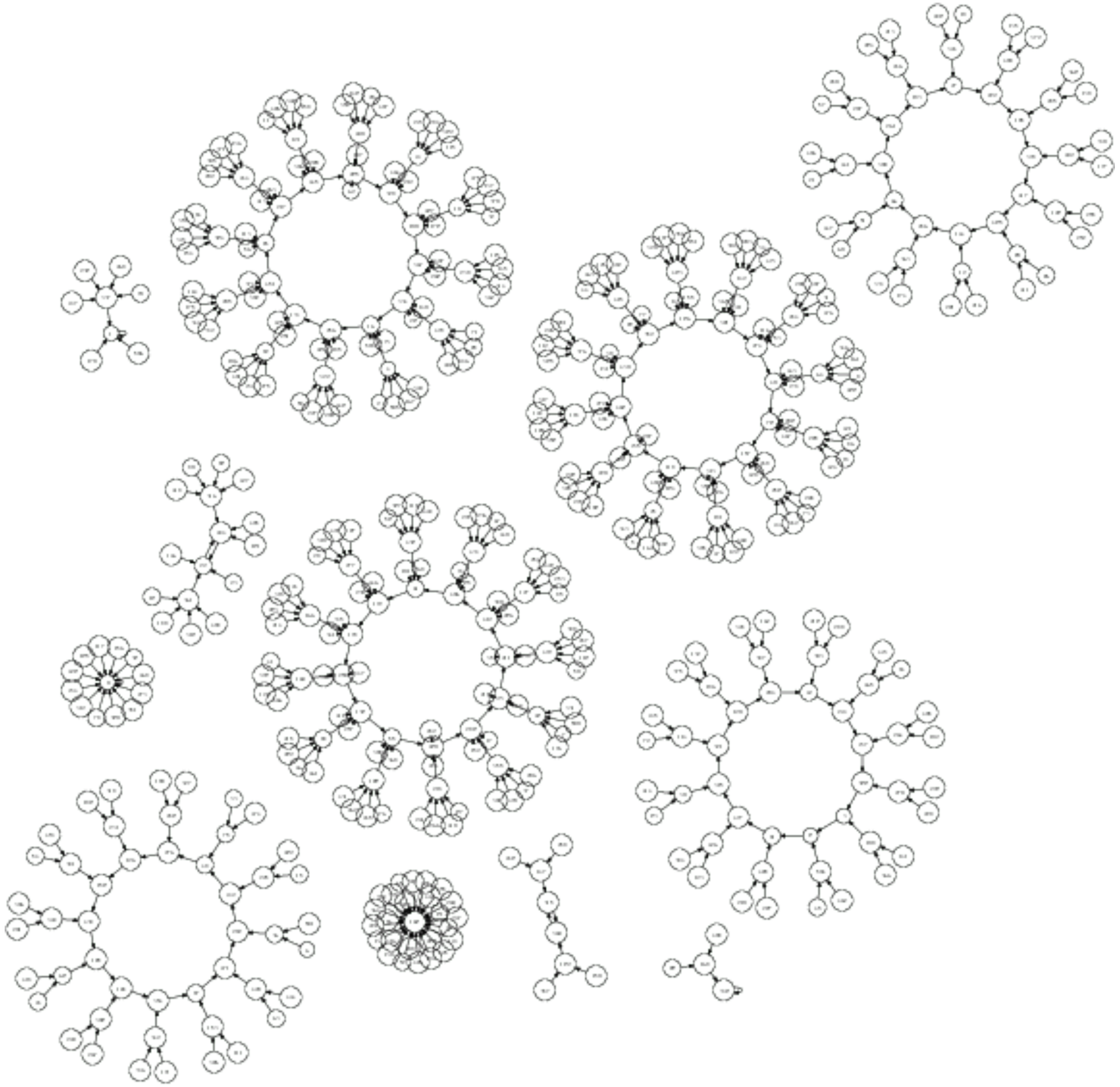
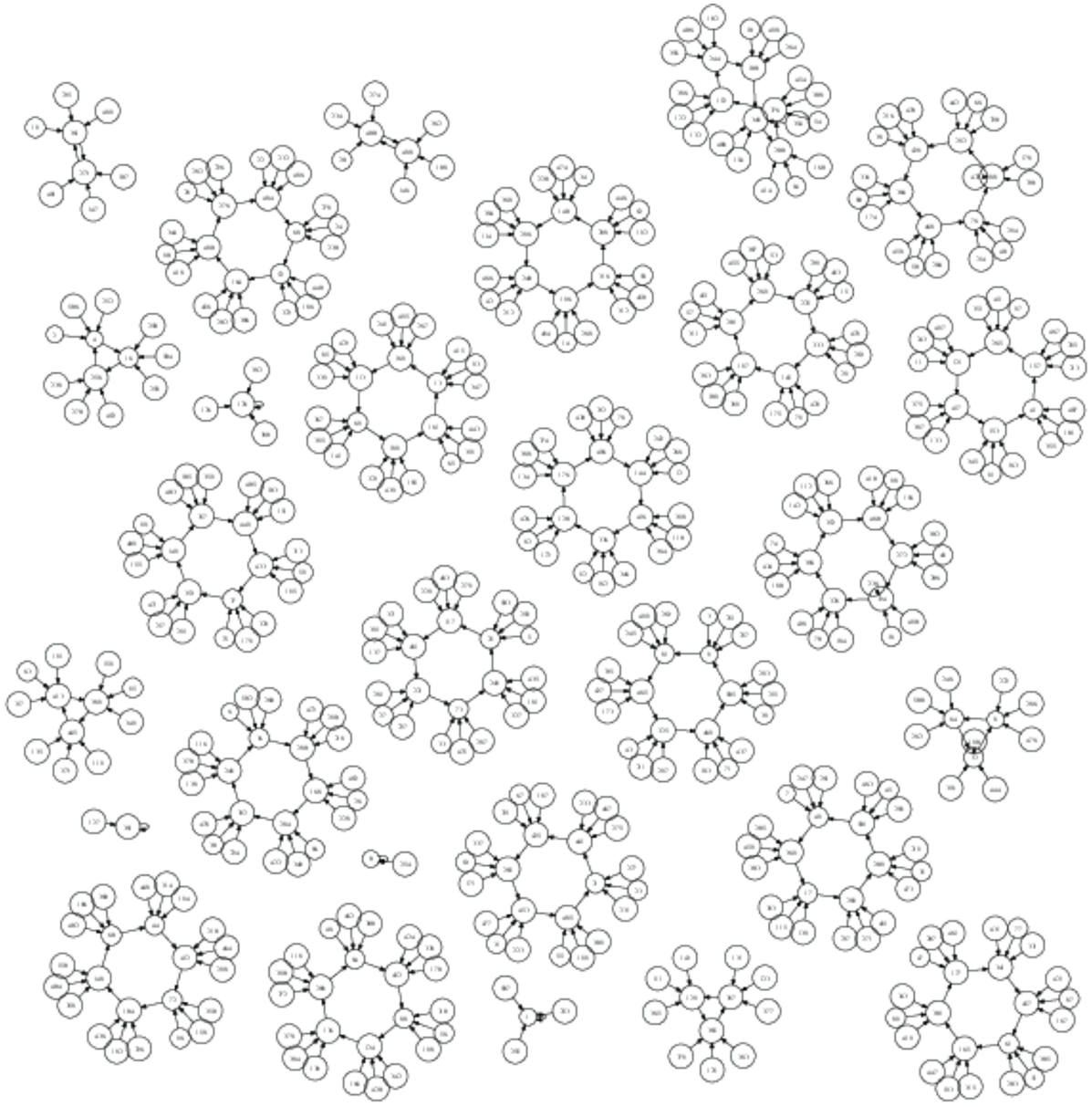
The iteration always ends with ‘1’.

No number leads to a subgraph that never connects to the main graph.

No sequence falling into a loop that does not terminate with 1.

Brian Hayes wrote an excellent article on 3x+1 in Jan 84 SciAm,

Computer Recreations, "On the ups and downs of hailstone numbers"

fin.

Sharing? Stories?