LangTominoes !?

JoHN MiLLeR

G4G 16

San Francisco

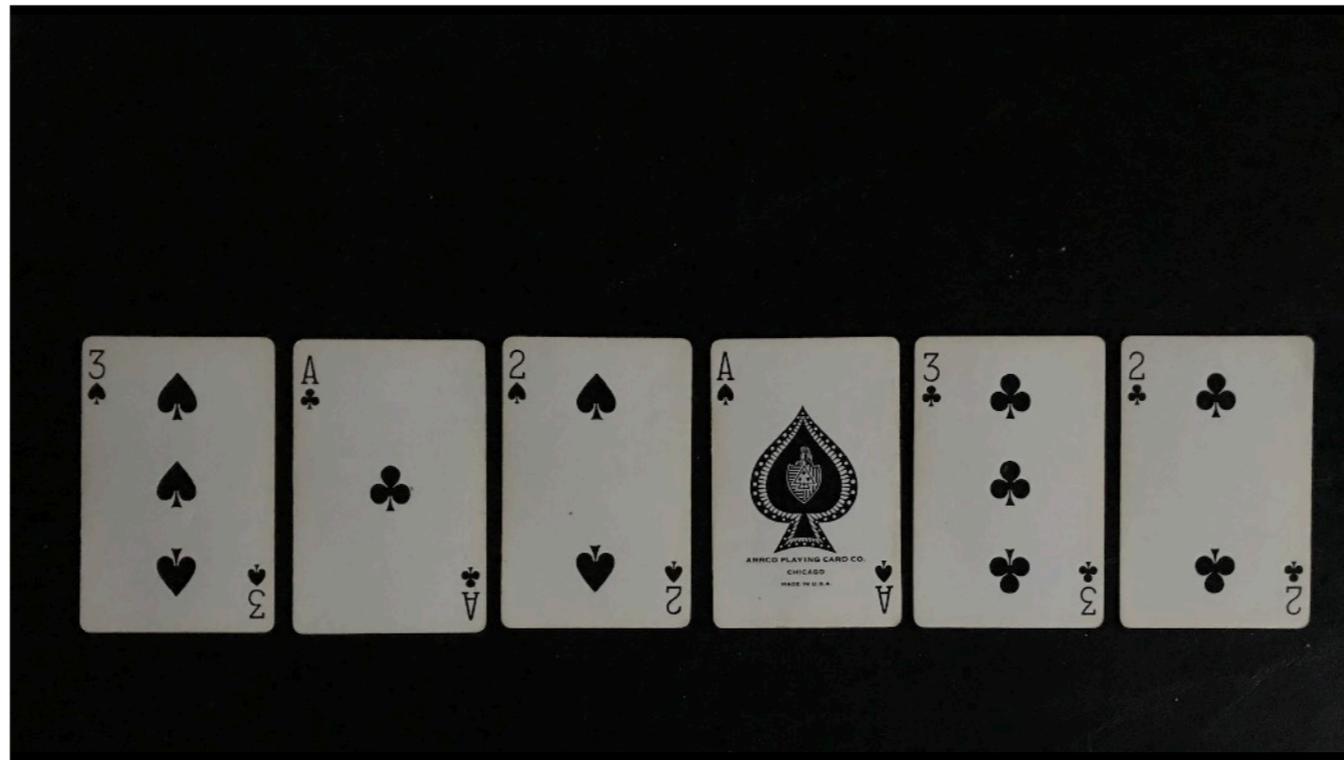Feb 22, 2026

dialectrix.com

HELLO ! Thank you.

—

Two dashes indicates notes for myself, in case needed for reference. They won't be read during the presentation.
To be clear - there is no time for questions at the end of any 6 minute talk. There will be ~170 presentations!
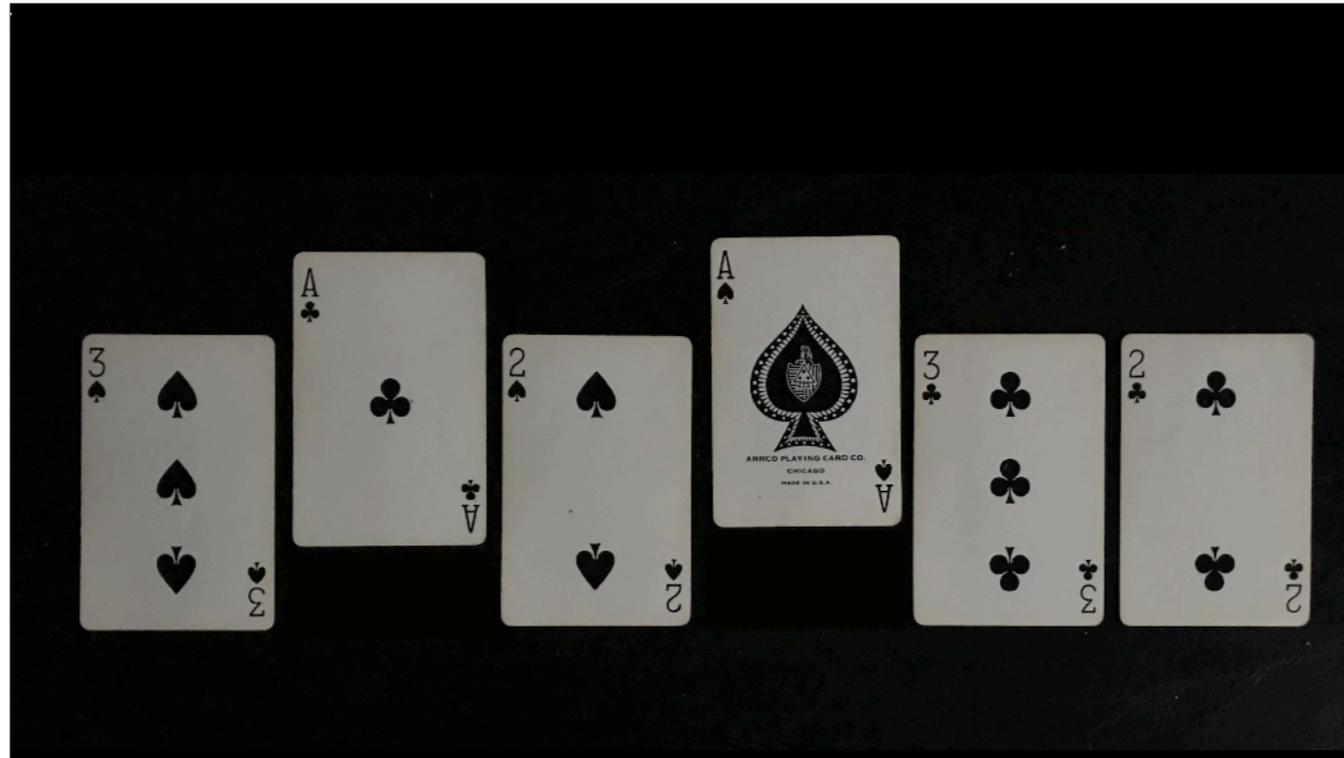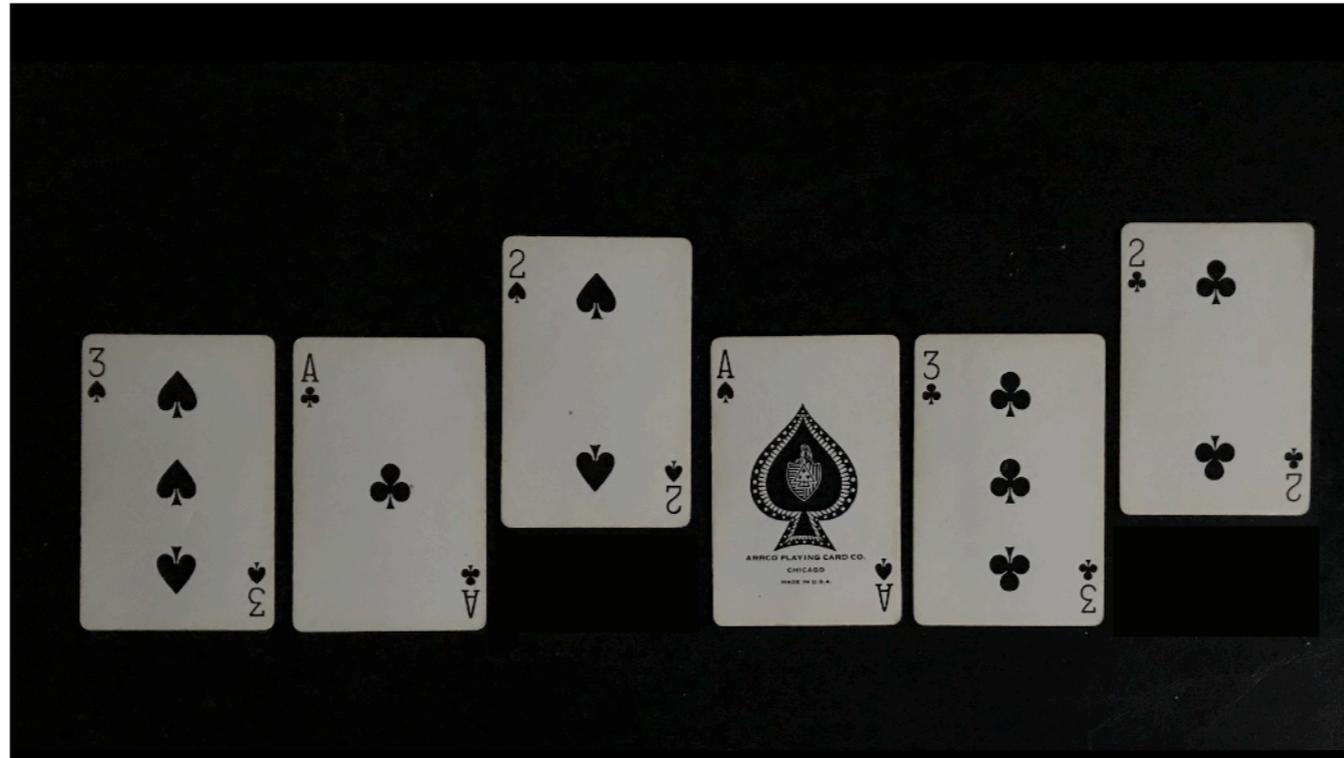
What is LANGFORD's PROBLEM ??
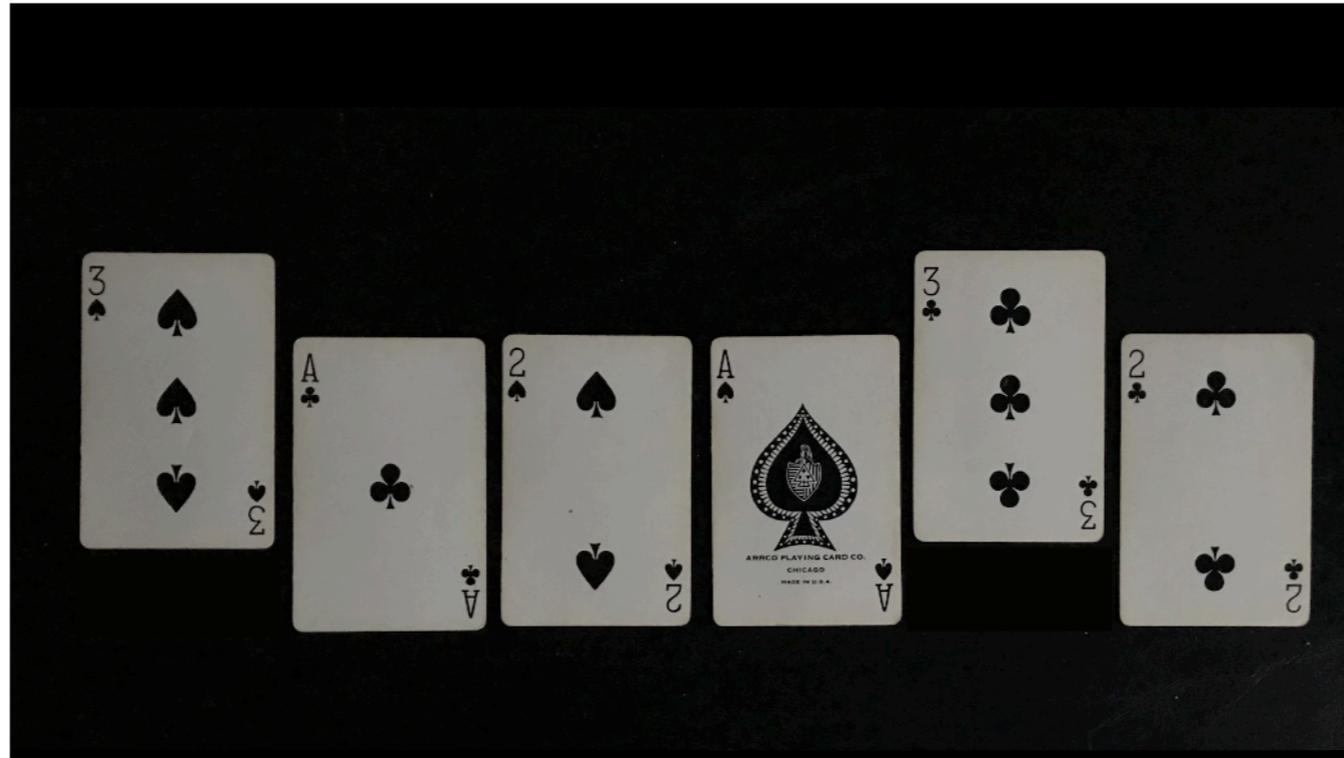
A Primer

So, first, a primer on Langford's Problem…

Let's consider 3 PAIRS of cards,
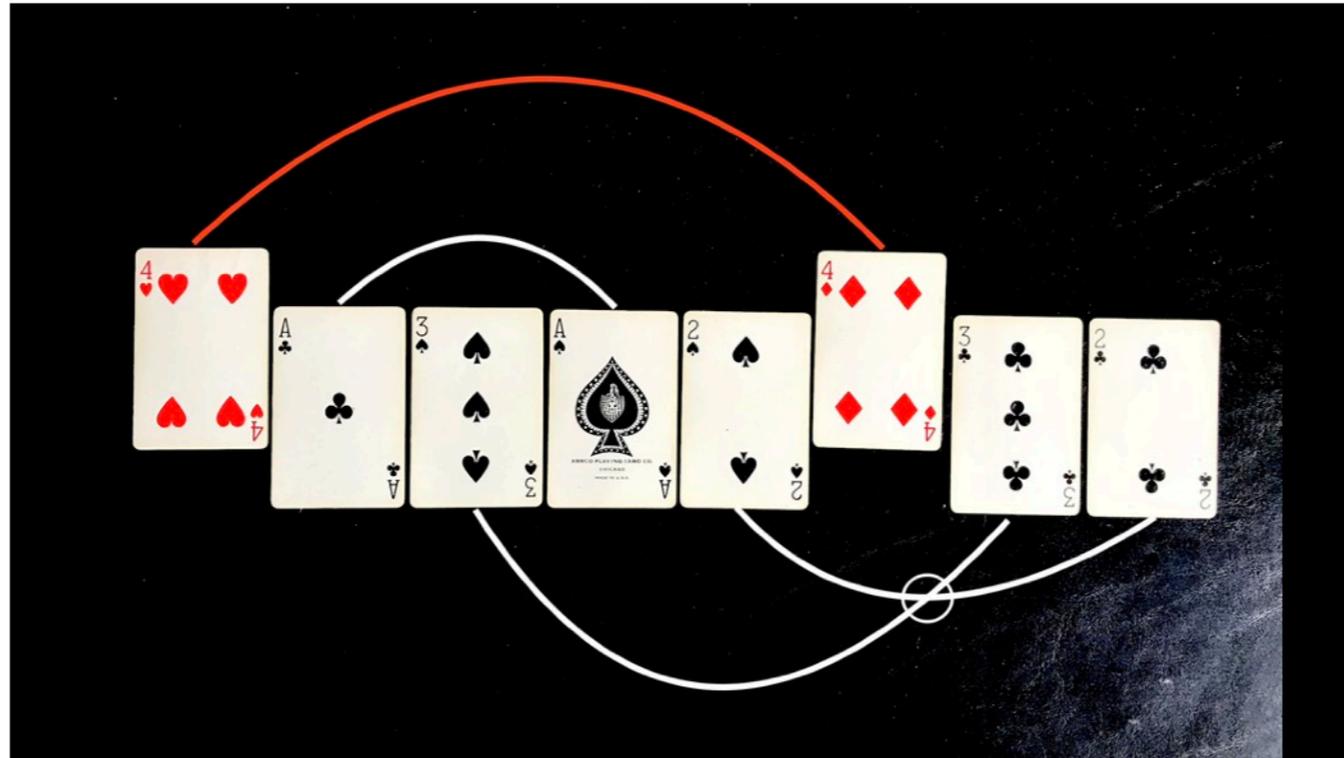
arranged so that …

One card is between the Aces;

and TWO cards are between the duces;

and Three cards are between the Treys.

That's the general idea.

Langford found that he could make an arrangement with 4 pairs, but not with 5 or 6 pairs.

It turns out, it can ONLY be done if the number (of pairs) is a multiple of 4, or one less:
For example…. 100 pairs!

Thus ends my Langford Primer…

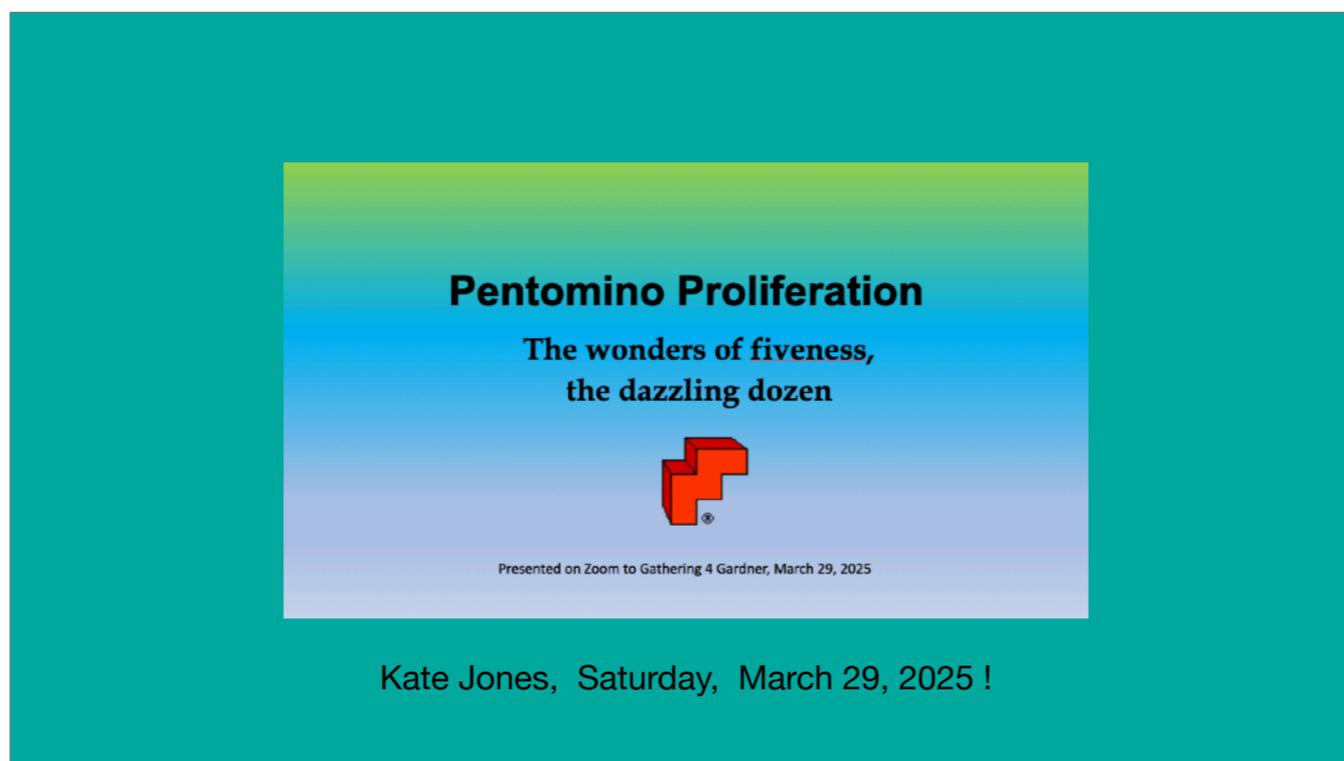BTW - Most such SUCH arrangements are "non-planar" — some lines connecting pairs cross, as shown here.
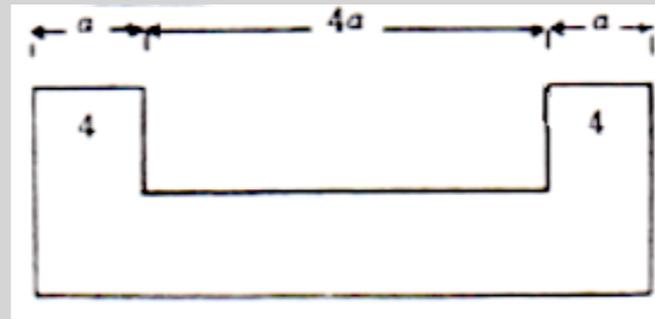
Langford's Jig
in 1958 Math Gazette

In 1958, Langford worked "by hand"…

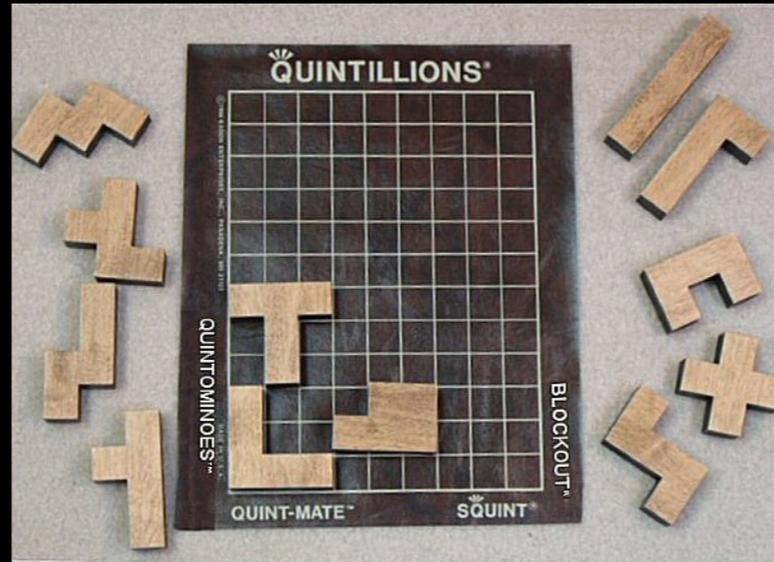cutting card stock in this SQUAT shape to "experiment" with pairs of numbers.

Kate Jones, Saturday, March 29, 2025 !

Last year - I Zoomed in on Kate's "Pentomino Proliferation".

I'd known about pentominoes since Martin covered them in October 1965.

Polycube version of Pentominoes
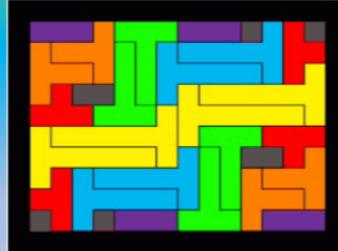
Kate Jones   GamePuzzles.com

Kate's "Quintillions" are poly-cubical pentominoes.

I got mine from her at G4G — But I never thought of using PolyCubes for Langford problem.

Robert's T Party

Aha!

At some point, Kate showed Robert's T Party !!

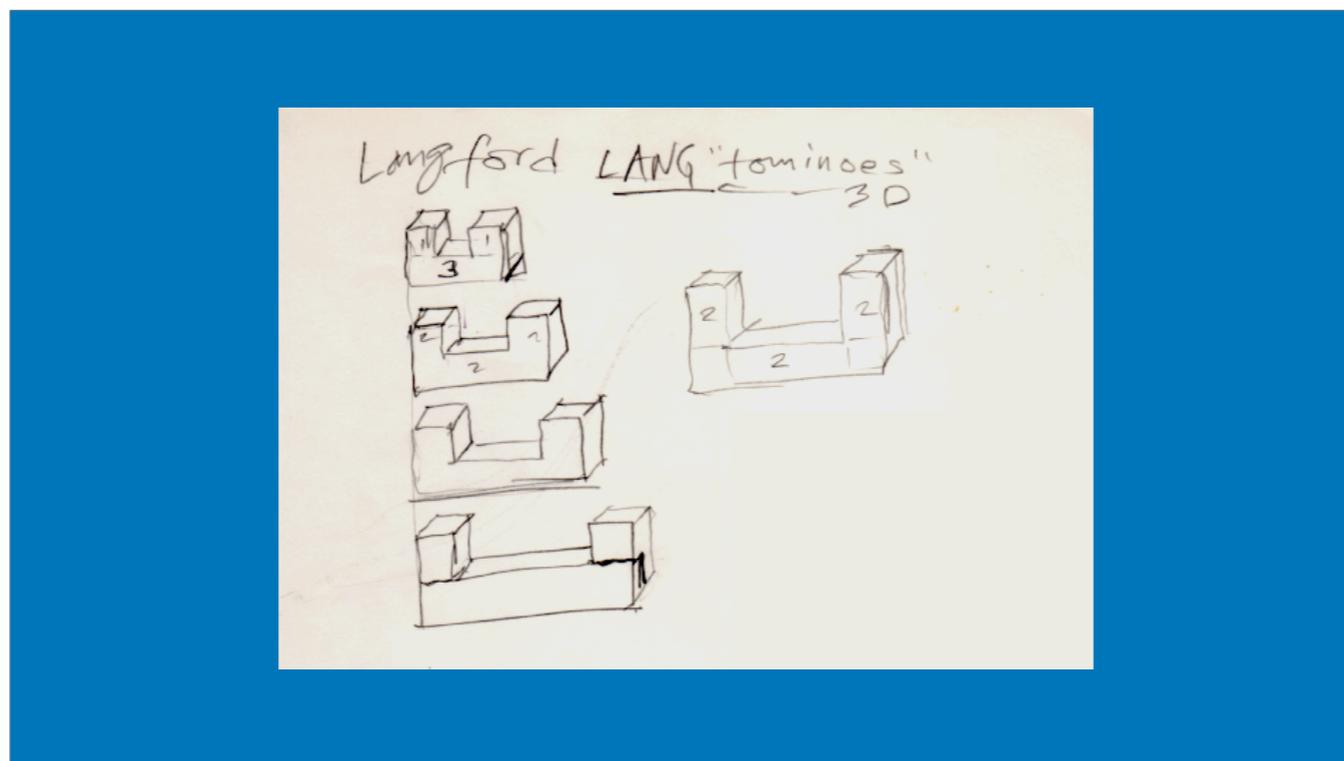The long T TILES jolted me.

 I thought, maybe I could do something similar for Langford's Problem.

—

There were T's of sizes 1, 2, 3, 4, 5-izes ! !

"Robert's T Party", by Robert Vermillion (LINK)
https://www.gamepuzzles.com/polycub4.htm#RTP

I started doodling…

and dubbed these solids "LangTominoes".
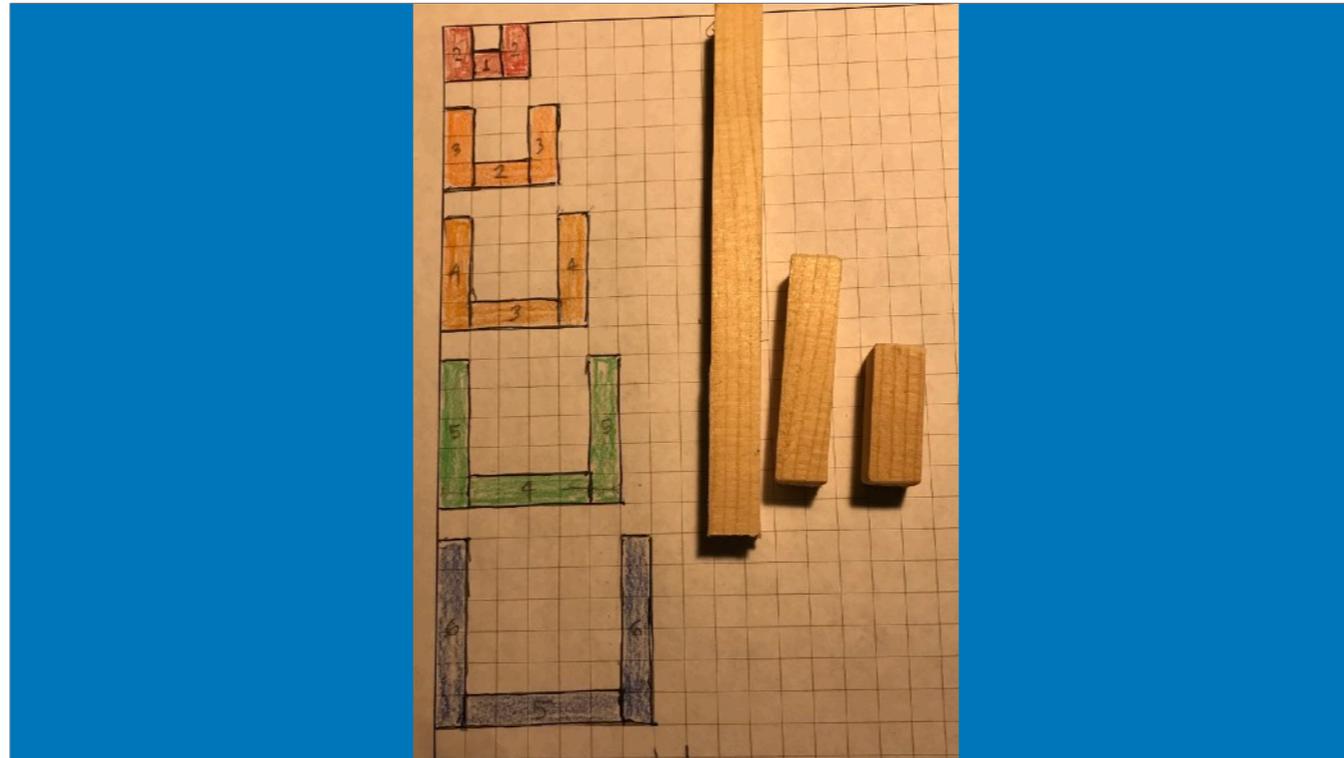
I thought TALLER wickets might work better than Langford's squat jigs.

—

(ignore Devil's Gate?)
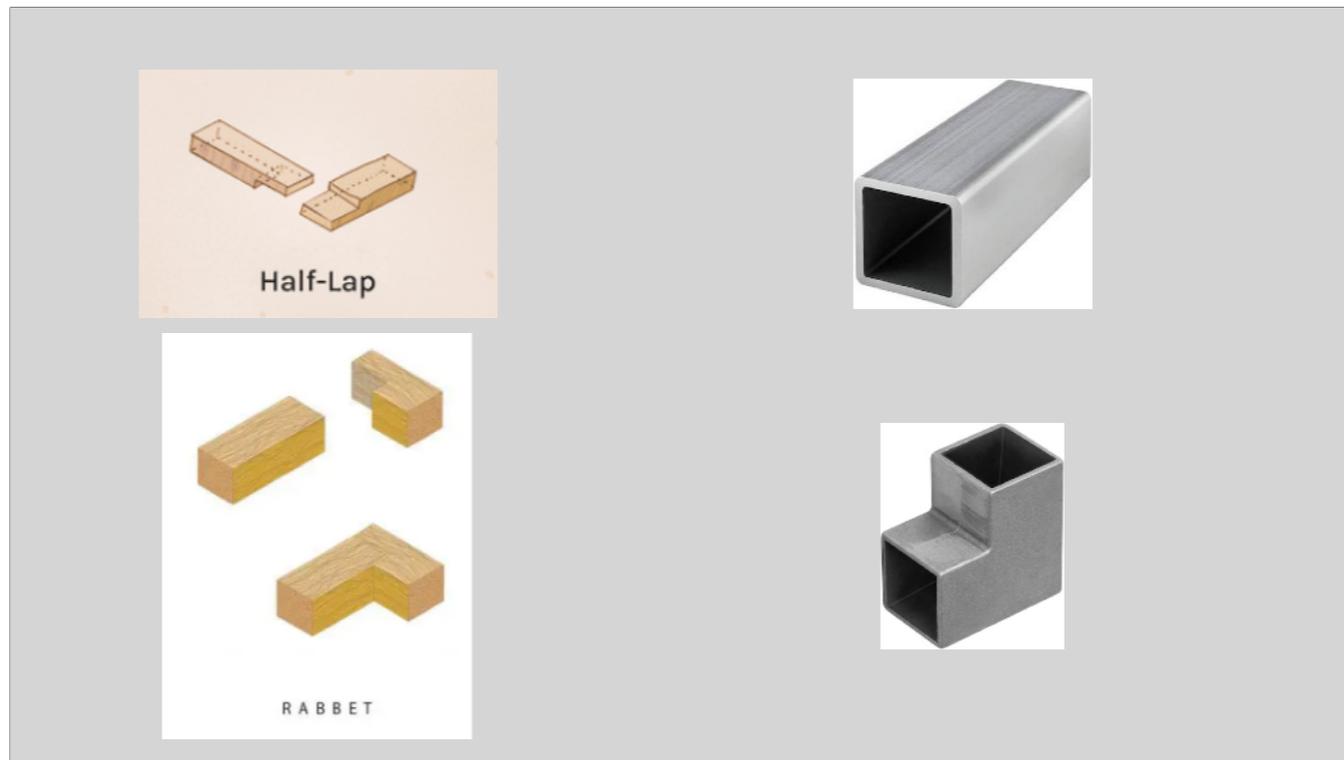
I considered various materials and joint designs.

—

Not laser-cut wood - afraid of leverage, breaking at corners.

Could I make Langford pieces up out of Wood,
requiring different kinds of joints??

Aluminum tubing with corner connectors seemed clunky (and $pendy)

Molding plastic pieces seemed obvious, but how to make all the molds??

Hexominoes on Etsy

"Items by CL"

But, during all this, I found and purchased a set of

Hex-Om-in-Oes on Etsy.

They were 3D-printed, based on 18 millimeter cubes.

So, I resolved to try printing some plastic!

OpenSCAD

```
1.  n = 3;

2.  union( ) {

3.    cube( [n+1,1,1] );

4.    cube( [1,n+2,1] );

5.    translate ( [0, n+1, 0] )

6.    cube( [n+1,1,1] );

7.  };
```
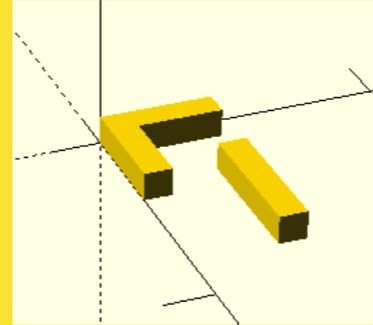
I'd never 3D-Printed anything.

I wrote to Bruce Oberg, who referred me to a Script-based CAD program.

Thanks Bruce!
—

Another way to model would be to have a rectangular prism and subtract a square prism from it.
I learned coding by trial & error!
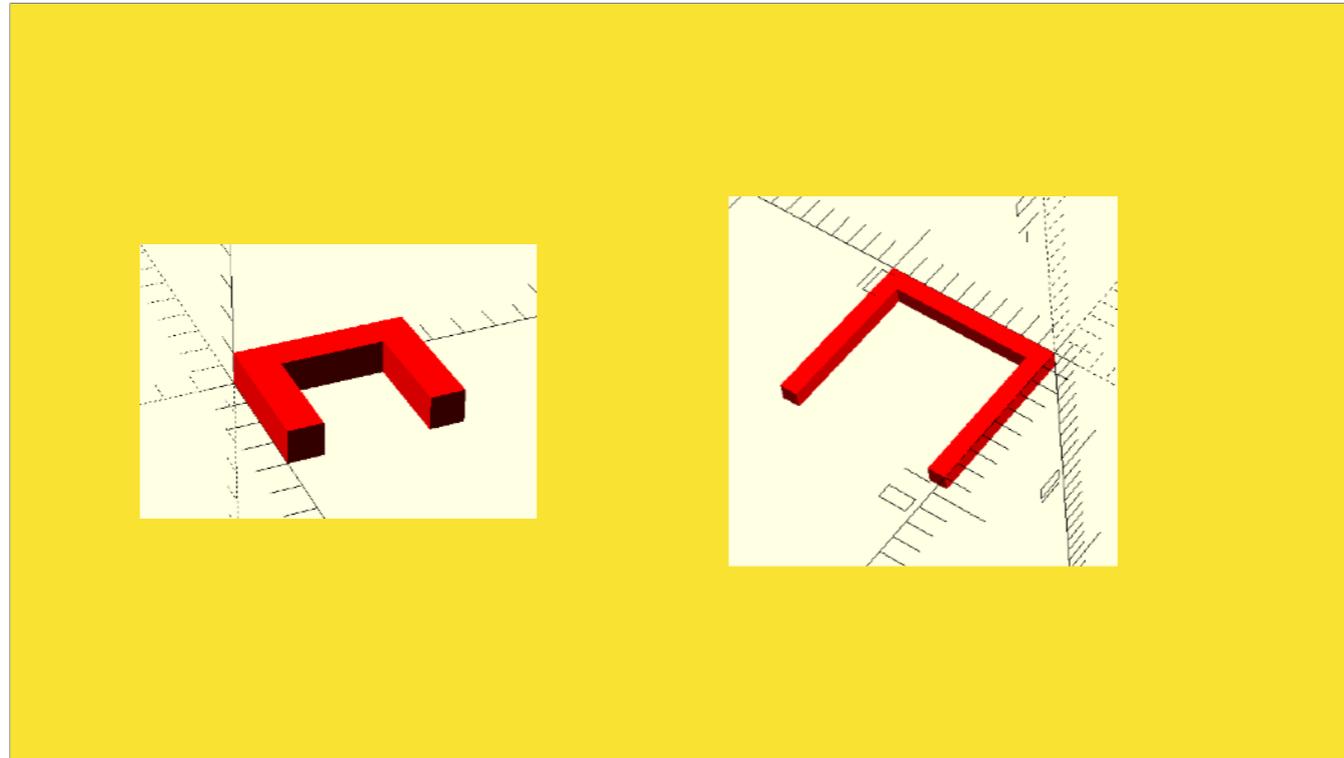
The model was trivial -

a base piece with two prongs.

—

I learned a few things

For example - it's not good to model sub-components as 'adjacent' — just touching.
The printer needs to know for sure that they are solidly joined.

So, I coded my object with over-lapping "corners" (Is that Bad?)

Anyway, I thought I would send files off to "SHAPEWAYS" for printing …

But I discovered there were local 3-D print shops!

Same with rendering - I saw Flicking on the boundaries.

Talus Design 3D in Portland took them on as a Challenge.
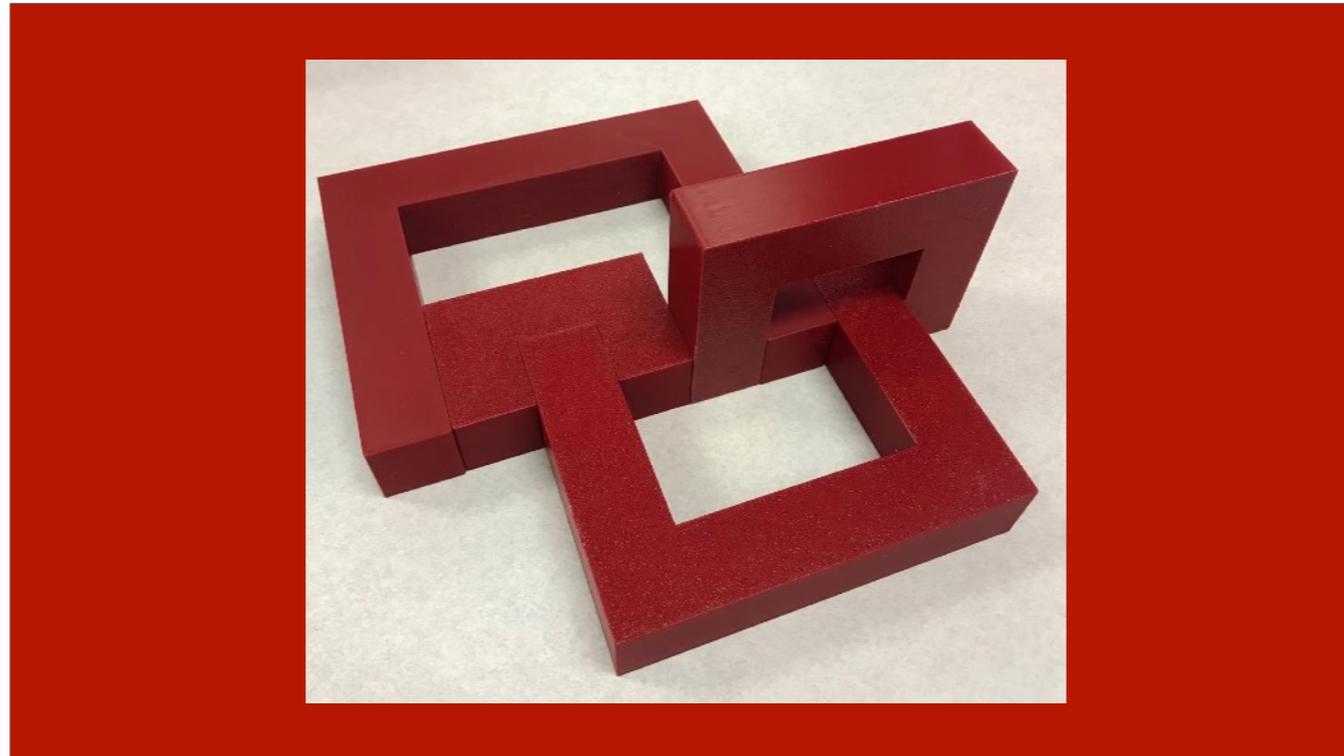
I had 1, 2, 3, 4 printed to see if they'd FIT together.

I didn't know if they'd be too tight, or what!

They fit nicely.

—

AHEAD, You'll see this arrangement done with 4 colors.

Question for me - did I anticipate arranging them in this manner? Most certainly, but I don't remember!
Check my notes!?

There was no point doing different colors, so I chose ALL RED.

I don't wanna explain what I'd have done CAD-wise if they were too tight!

BTW - Each of these took about 3 hours to print!

EVENs   ODDs

Printed by Talus Design 3D, Portland
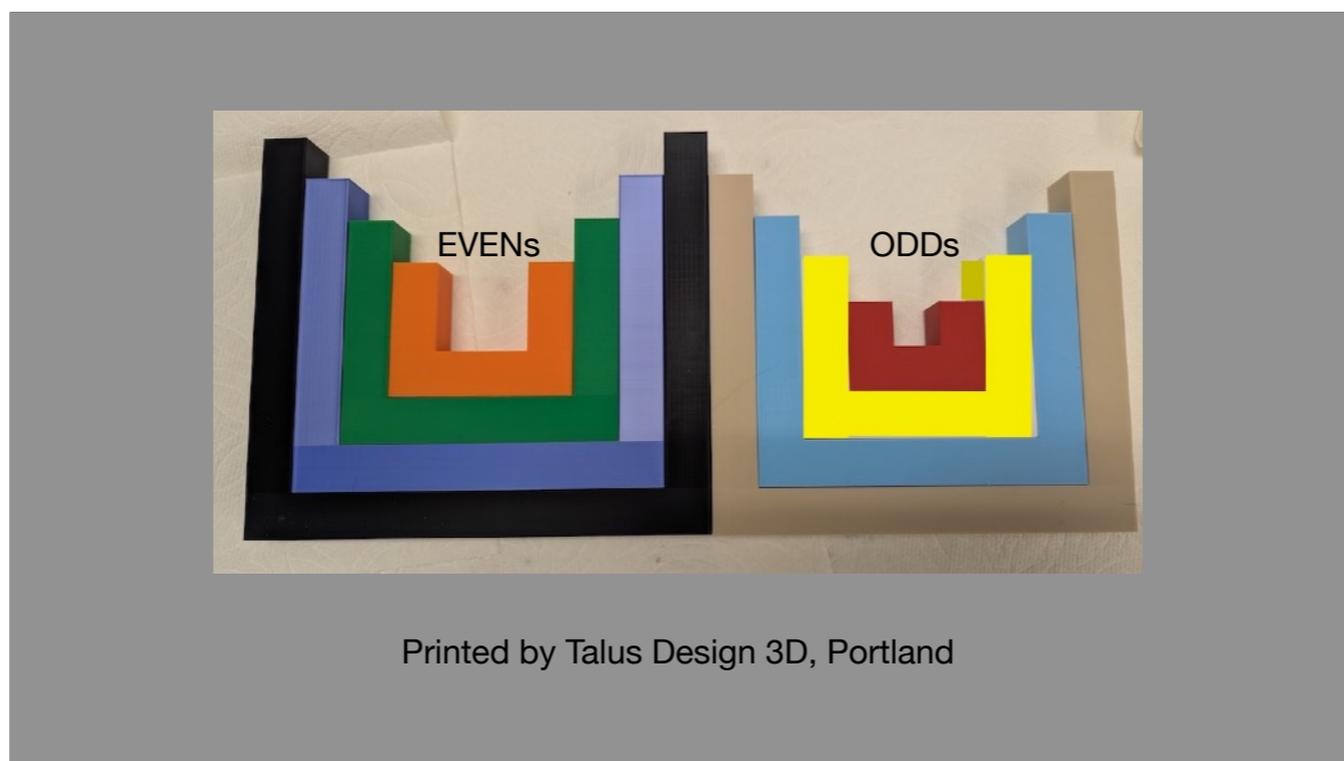
SO, I had LangTominoes 1 thru 8 printed, in colors, at size 18mm.

Nice, but SPENDY!

so, I ordered 10 sets from the HexOmino guy to share with people, including Kate and Don!

You can order a set, or print your own. See my web site.

—

I have this set aside for Don K-Nuth.
~~ $22 for a set (plus shipping)
Customs + shipping alone takes two weeks!

One of Knuth's planar solutions for n=8
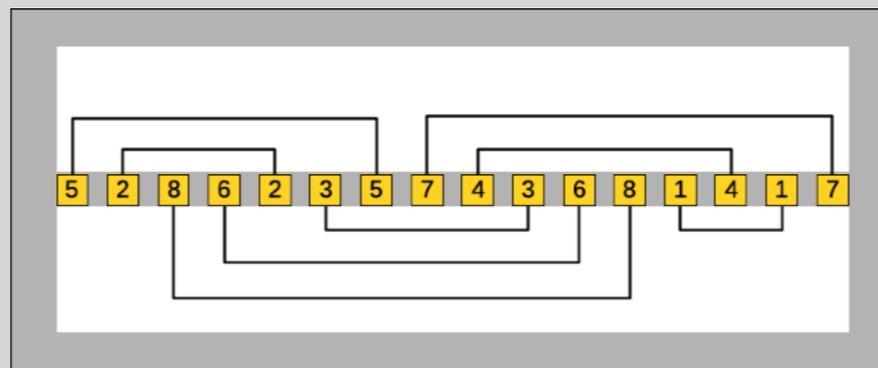
We must include this figure for 8 pairs with non-crossing lines, from Ka-nuth's Volume 4, Algorithms.

Planar arrangements are rare — Only FOUR of the 150 solutions for "8" are planar.

7 pairs have NONE!

—

[Q: Were Planar Solutions written about earlier? Most likely! Check Knuth refs.]
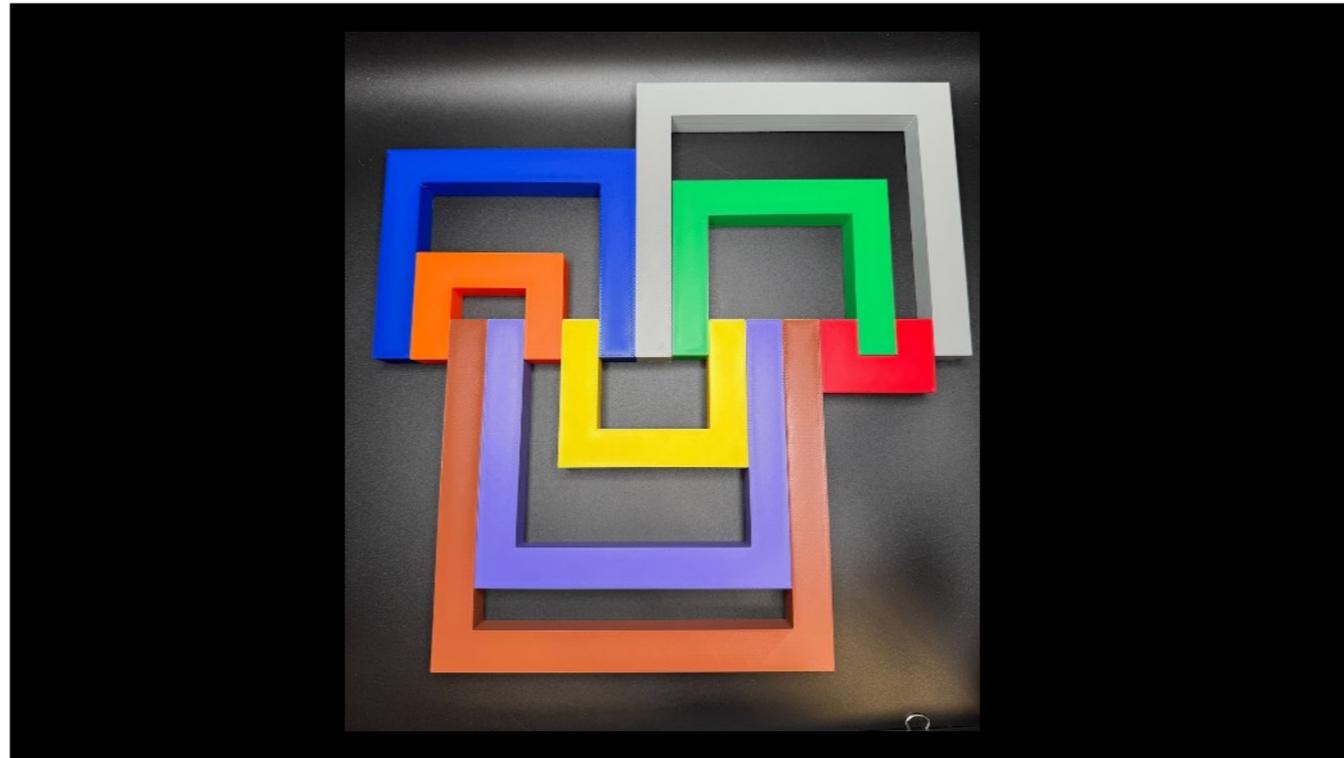
Here's that same planar Solution.

—

5 2 8 6 2 3 5 7 4 3 6 8 1 4 1 7

11, Planar

Here's 11.

EVEN IF a particular solution is PLANAR, you Still have to figure out which prongs point Up and which point Down.
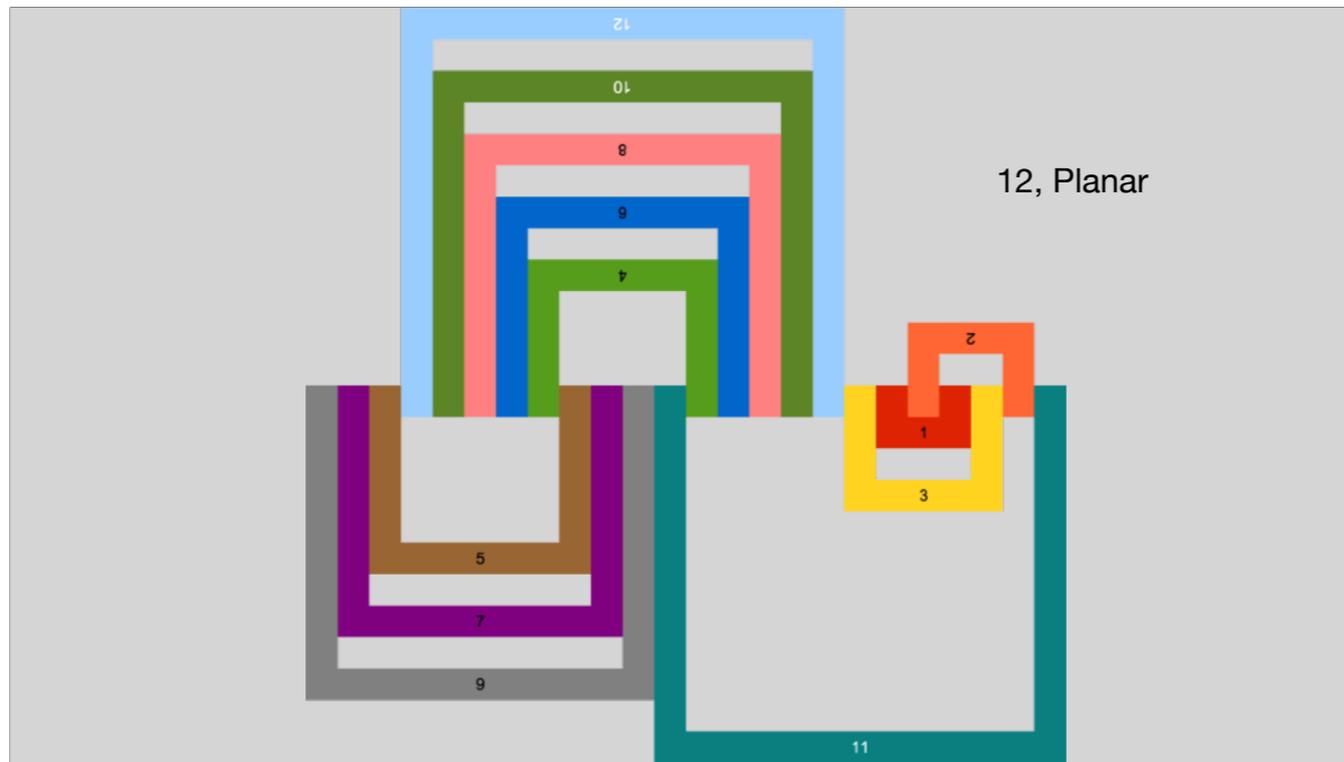
12, Planar

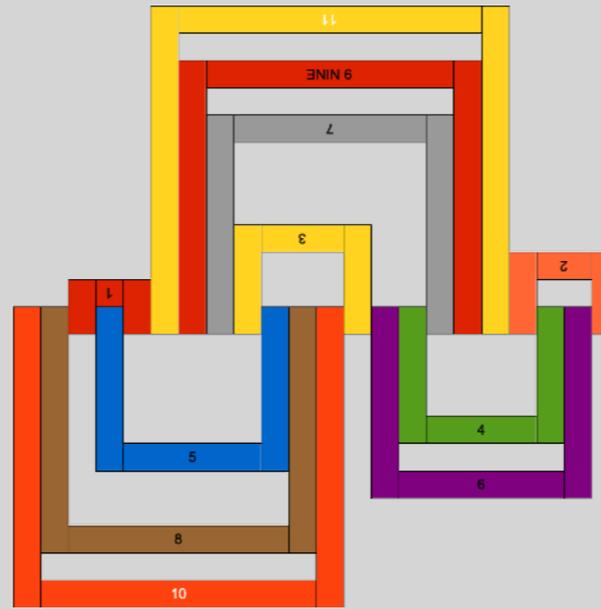Here is one of the 40 Planar arrangements for 12.

It seems "obvious" that PLANAR solutions can be realized with LangTominoes.

(Maybe YOU can re-solve The Prongs Up/Down question.)

16, Planar

G4G 16 !
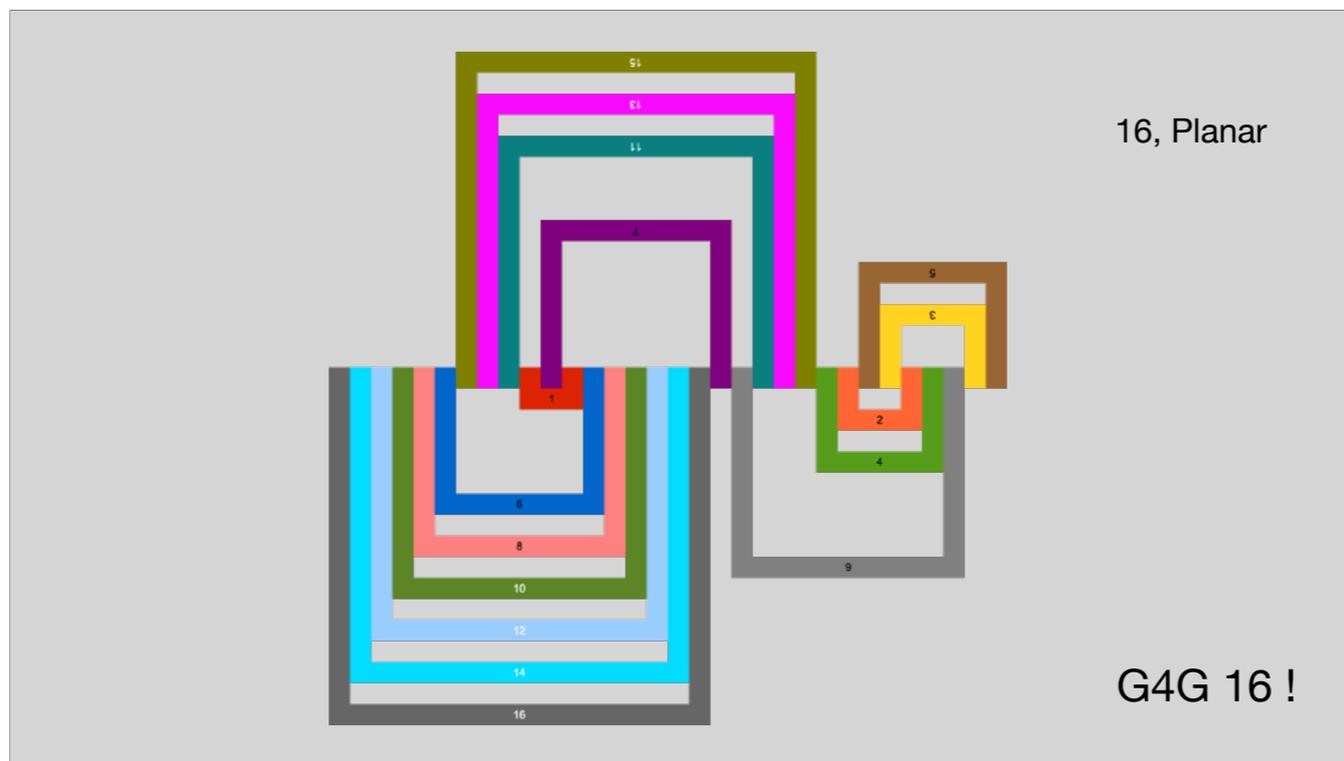
Here's my Obligatory 16 !

Imagine strips of *glass* laid out like this, and fused together.

—

There are 274 unique planar arrangement for n=16.

Here is the first, (in lexicographic order) solution to 16.

FOUR of these 8 LangTominoes are 'standing up'.

In this configuration,
those four could just lie back flat on the table, no problem.

However, the N=4 solution is unavoidably non-planar…

But! — LANGTOMINOES can stand on their Toes!

This shows a "Jumper", jumping out of the Plane.

RED is the 1.
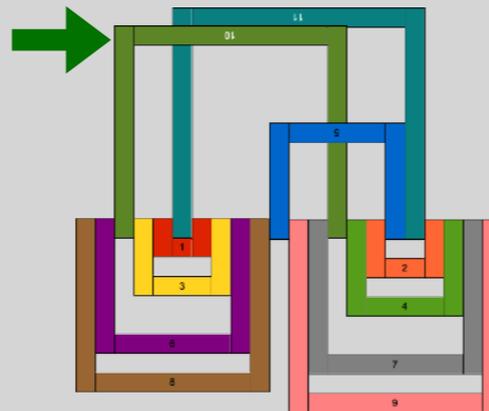ORANGE is 2.

And so on.

—

WHITE is 3.
GREEN is the 4.

Since these fit tightly, you can pick this up, gently. But a larger assembly won't hold together.

(btw - This is the same arrangement as the All Red prototypes I showed earlier)

We discussed ideas for that during a Saturday Zoom talk last summer.

a Roy O Davies Construction

8 6 10 3 1 11 1 3 6 8 5 9 7 10 4 2 5 11 2 4 7 9
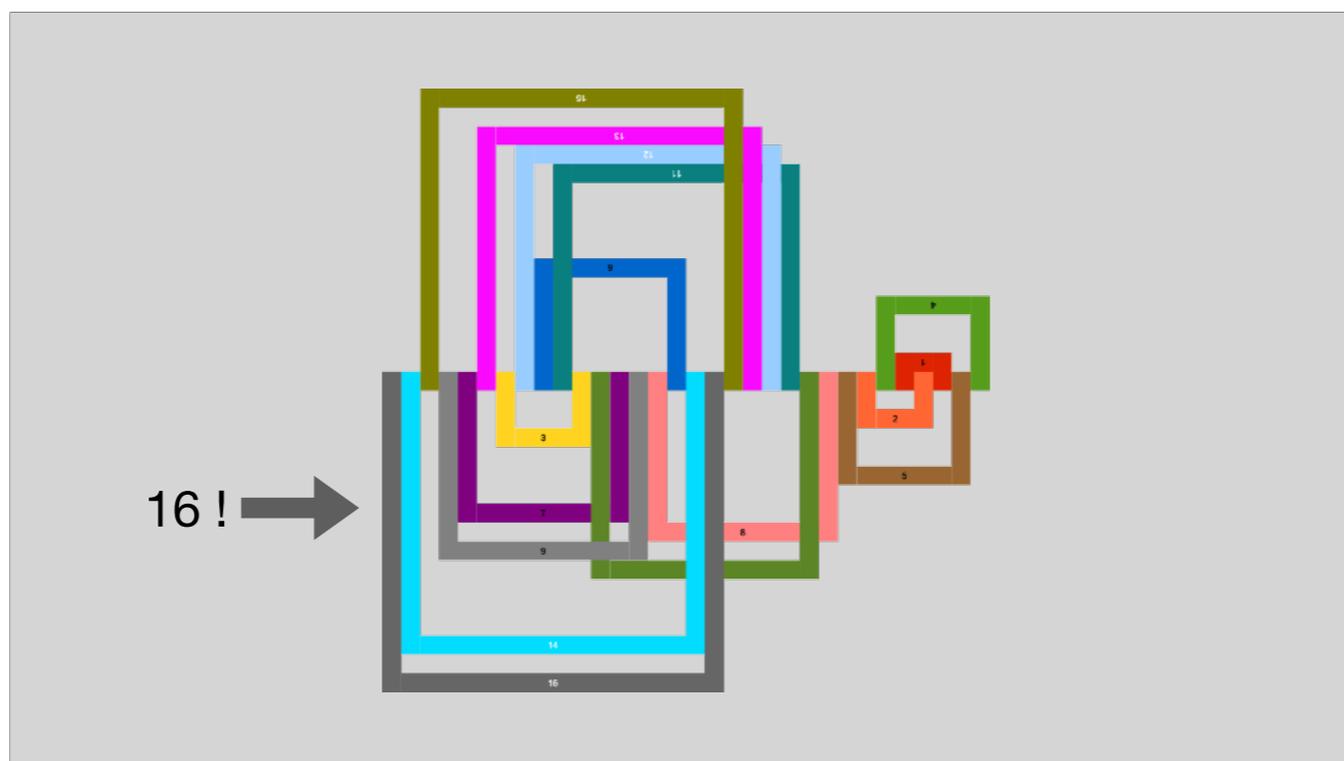
All Davies constructions are Non-planar.

That Green 10 crosses the 5, and the 11.

But we can stand that 10 up -- to Jump over both crossings!
—

Standing green #10 up on its prongs allows a '3D planar' solution.

16 !

Another 16…

Non-planar, and tangled !

—

Can you flip certain ones toward you
and other ones //away// from you
to make this '4D' Planar?

Can we decompose this into two orthogonal arrangements? I'll wager - not always.

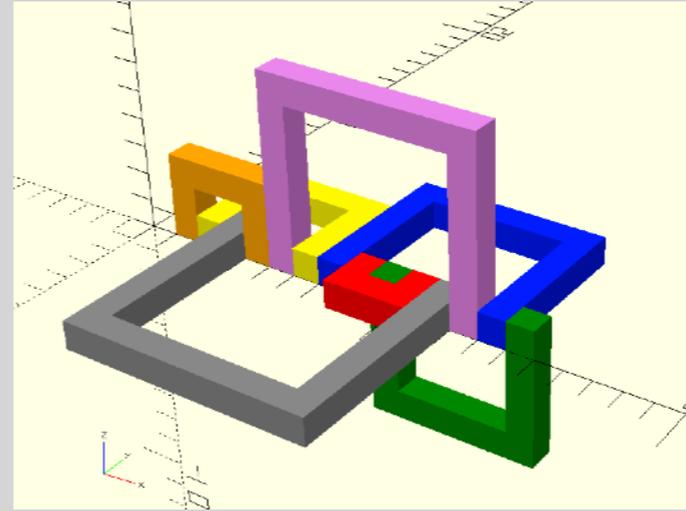[There are 254 flat (2D) planer solutions]   I don't have a list!

G4G 17 -- Can I do a 16 arrangement as a LangTomino Solid. NOT before Feb 18th!!

Research Questions

Imagine 4 semi-planes, or "VANES".

Q1:
How many vanes would be required for a given N, so that each vane contains only planar connections?

Q2:
Would 4 vanes suffice?

The Big Question is… Can We 'Do' all solutions with LangTominoes?If not, how many Can We Do for a given N?

(((read off slide)))

BTW, we need a Notation for LangTomino arrangements.
—

Exercise: Construct all 26 solutions for n=7 with LangTominoes.

Can you tell how many vanes would be needed, simply by scanning a given solution Left-to-Right?

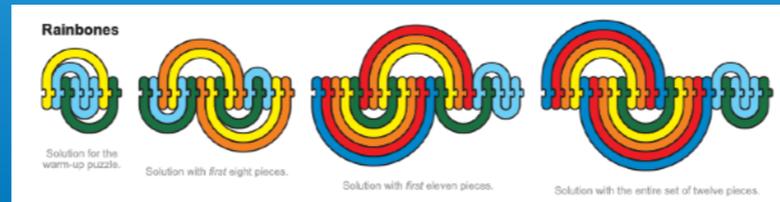One needs an algorithm and data structure for programming.

What can YOU do?

- Buy a set of 8 LangTominoes from Items By CL

- Download the STL files and print all you want.

- Visit dialectrix.com —> langford —> LangTominoes.html

fin ??   *it's only just a beginning!!*

That's it! For more on Langford's Problem, find my web page.

Any Time Left? Here are some slides for fun…

George Miller and Don Knuth created 'Rainbones'.

They connect nicely with each other, unlike LangTominoes V1.0

However, Rainbones must lie flat on the plane — They can't stand up.

—

Stan Isaacs used this puzzle as his exchange gift at IPP27
 in 2007 in Gold Coast, Australia.

PAUL KLEE - With the Rainbow - 1917

DIALECTRIX.com

A little cubism for you…

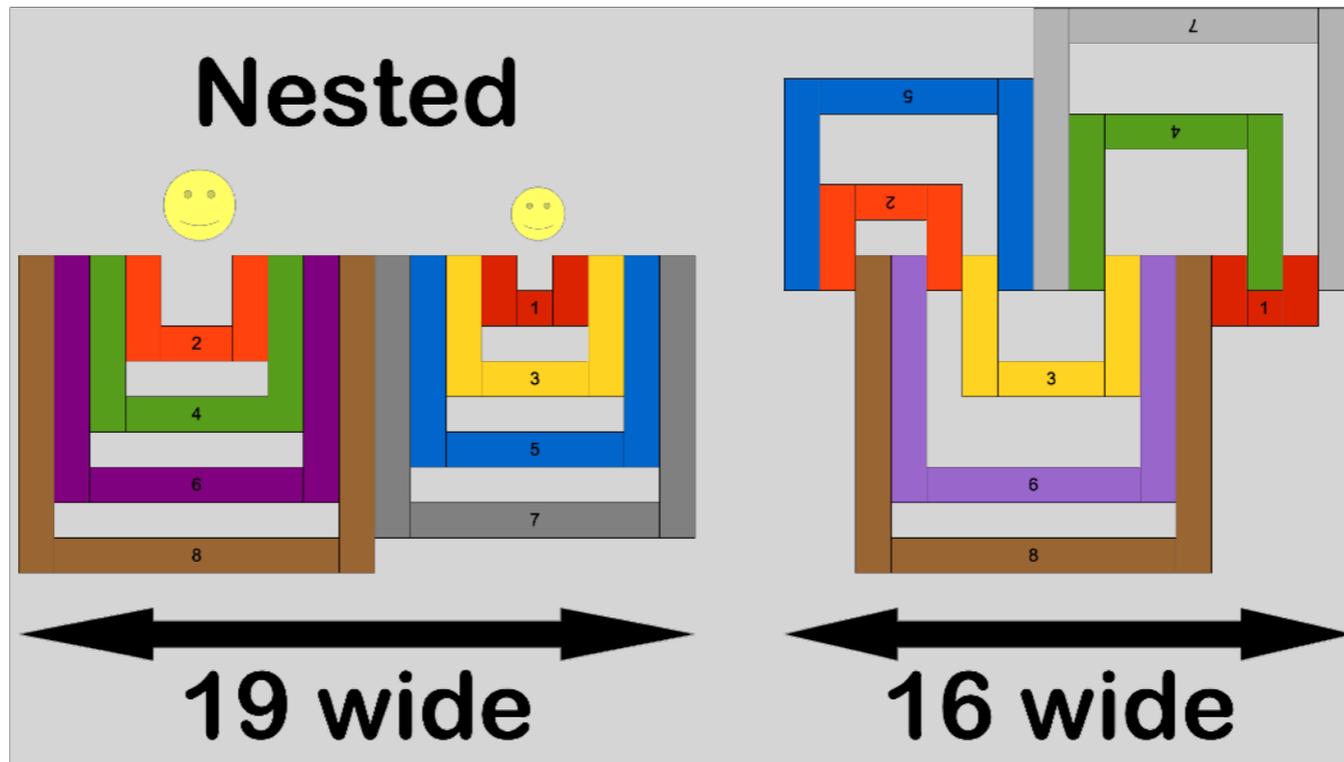The Logo for Langford Construction is made from 'L' and 'C' to make a little house!

—

https://builtbylcs.com/home/about/

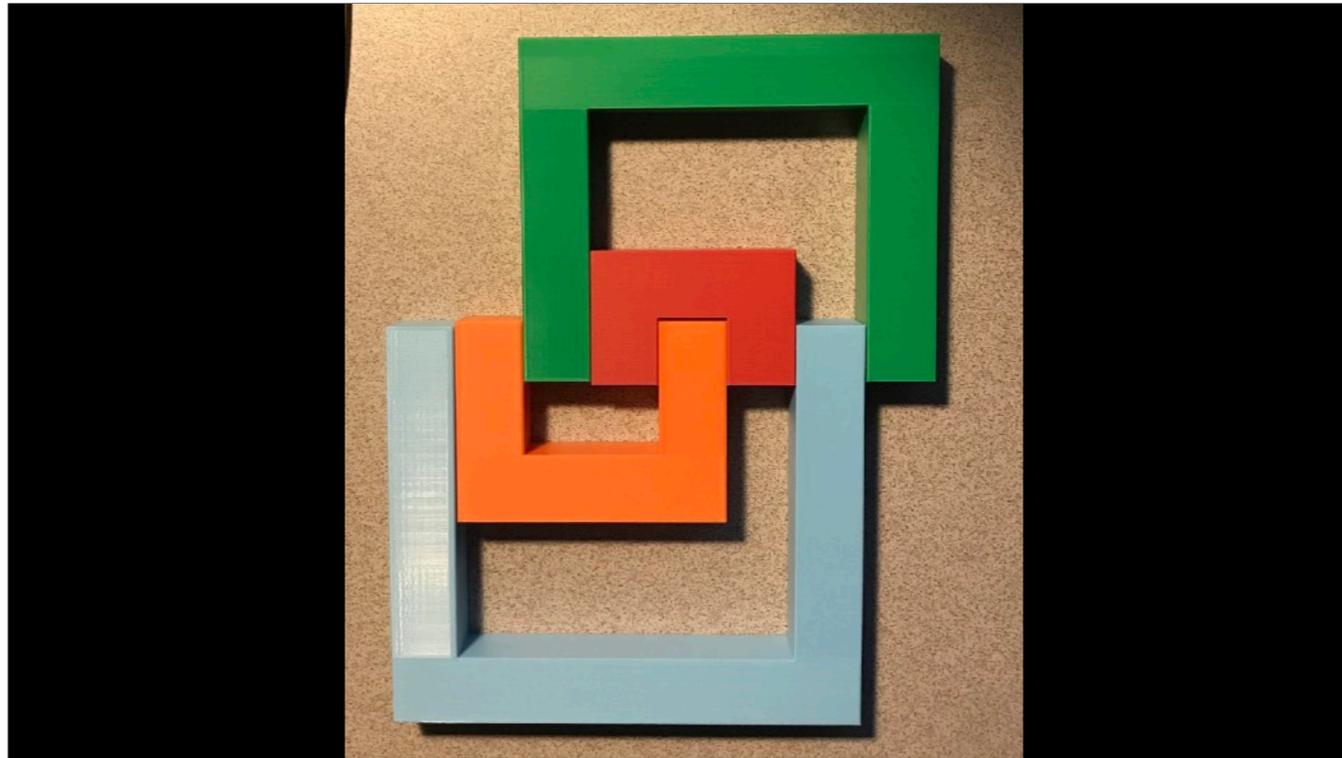"Zeichen" by

Gerhard Hotter

Artist Gerhard Hotter's recent exhibition "CODES" had this artwork.

A curious fact.
The nested even & odd arrangement is 19 units wide,
whereas
the planar arrangement on the right is 16 wide, …

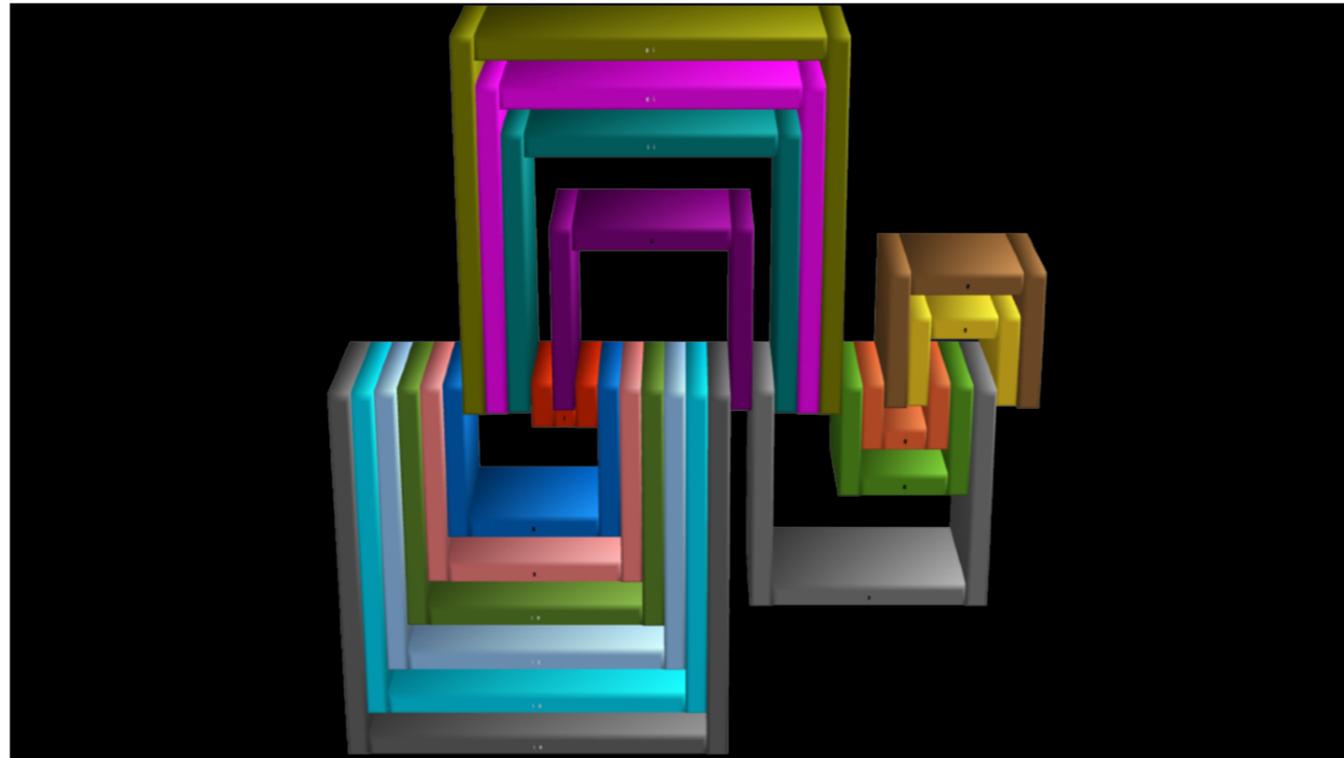BECAUSE the nested set has two empty gaps. (The Smilie Faces)

Here is an arrangement of [5,2,4,1]

—

Knuth challenge with Rainbones….   do 2 + 3 + 4 +5

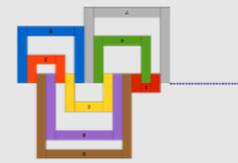Polylactic Acid (PLA) is one of the most popular plastics used for 3D printing.

A little Eye Candy here for 16.

"Langtominoes" by JoHN MiLLer

Game board for the four planar solutions of Langford's Problem for n=8.

www.dialectrix.com/langford/LangTominoes.html

| 5 | 2 | 8 | 6 | 2 | 3 | 5 | 7 | 4 | 3 | 6 | 8 | 1 | 4 | 1 | 7 |

| 1 | 4 | 1 | 8 | 6 | 3 | 4 | 7 | 5 | 3 | 2 | 6 | 8 | 2 | 5 | 7 |

| 4 | 2 | 7 | 5 | 2 | 4 | 8 | 6 | 3 | 5 | 7 | 1 | 3 | 1 | 6 | 8 |

| 1 | 3 | 1 | 7 | 5 | 3 | 8 | 6 | 4 | 2 | 5 | 7 | 2 | 4 | 6 | 8 |

Game board to help beginners set up the 4 the planar solutions for 8.

Question: Might there be a Two-player Game using one or two sets?